South Sudan

ALP 4

Mathematics

Learner's Level 4

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FOREWORD

I am delighted to write the foreword for this book. The Ministry of General Education and Instruction (MoGE&I) has developed the Accelerated Learning Programme (ALP) textbooks based on the National Curriculum of South Sudan.

The textbook was written to help learners develop the background knowledge and understanding in the subject. It is intended largely to serve as a source of knowledge and understanding of the subject concerned, but not to be considered as a summary of what learners ought to study.

The National Curriculum is a competency based and learner-centered that aims to meet the educational needs and aspirations of the people of South Sudan. Its aims are manifold: (a) Good citizenship (b) successful lifelong learners, (c) creative, active and productive individuals; and (d) Environmentally responsible members of our society.
This textbook was designed by subject panelists to promote the learners'attainment of the following competencies; critical and creative thinking, communication, cooperation, culture and identity.

No one can write a book of this kind without support from colleagues, friends and family. Therefore, I am pleased to register my thanks to Dr Kuyok Abol Kuyok, the Undersecretary of the Ministry, who emphasized the importance of Alternative Education System (AES) and approved the development of its textbooks.

I also want to record my thanks toUstaz Omot Okony Olok, the Director General for Curriculum Development Centre (CDC) and Ustaz Shadrack Chol Stephen, the Director General for Alternative Education Systems (AES) who worked tirelessly with thesubject panelists to develop the textbooks.

Lastly, but not least, my greatest thanks and appreciation must go to the Global Partnership for Education (GPE) and UNICEF-South Sudan for without their support and partnership this textbook would not have seen light.

Hon. Awut Deng Acuil, MP
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UNIT 1: NUMBERS

1.1 Factors and Multiples

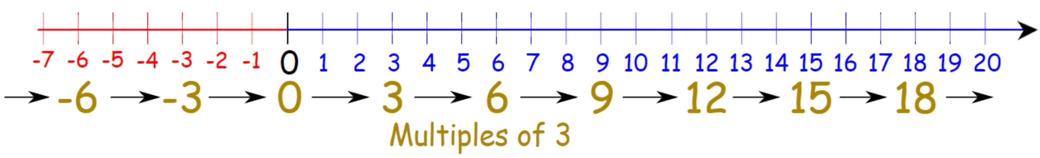
Factors and multiples are different things but they both involve multiplication:

Multiples

A multiple is the result of multiplying a number by an integer (not a fraction).

Example 1.

Multiples of 3:



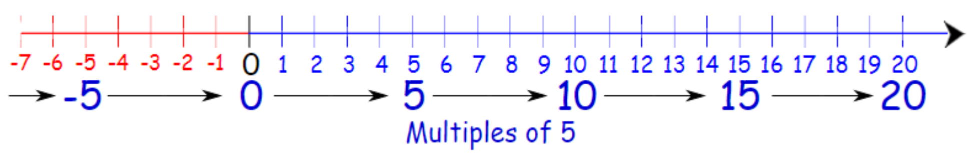
$$\ldots, -9, -6, -3, 0, 3, 6, 9, \ldots$$

15 **is** a multiple of 3, as $3 \times 5 = 15$

16 is **not** a multiple of 3

Example 2.

Multiples of 5:



$\ldots, -15, -10, -5, 0, 5, 10, 15, \ldots$

10 **is** a multiple of 5, as $5 \times 2 = 10$

11 is **not** a multiple of 5

Factors are what we can multiply to get the number.

Multiples are what we get after multiplying the number by an integer (not a fraction).

For example: the positive factors, and some multiples, of 6:

Factors:

$1 \times 6 = 6$, so **1** and **6** are factors of 6
$2 \times 3 = 6$, so **2** and **3** are factors of 6
Multiples:
$1 \times 6 = 6$, so **6** is a multiple of 6
$2 \times 6 = 12$, so **12** is a multiple of 6 and so on
(Note: there are negative factors and multiples as well)
Here are the details:
Factors
"Factors" are the numbers we can **multiply together** to get another number:

$$\underset{\text{Factor}}{2} \times \underset{\text{Factor}}{3} = 6$$

2 and 3 are factors of 6

A number can have **many** factors.

Example 3.

$3 \times 4 = 12$, so **3** and **4** are factors of 12

Also $2 \times 6 = 12$, so **2** and **6** are also factors of 12,

And $1 \times 12 = 12$, so **1** and **12** are factors of 12 as well.

AND because multiplying negatives makes a positive, $-1, -2, -3, -4, -6$ and -12 are also factors of 12:

$(-1) \times (-12) = 12$

$(-2) \times (-6) = 12$

$(-3) \times (-4) = 12$

So ALL the factors of 12 are:

1, 2, 3, 4, 6 and 12
AND **−1, −2, −3, −4, −6 and −12**

All Factors of a Number

Factors are the numbers you multiply together to get another number:
There can be many factors of a number.
Example: All the factors of **12**
$2 \times 6 = 12$,
but also $3 \times 4 = 12$,
And of course $1 \times 12 = 12$.
So **1, 2, 3, 4, 6 and 12** are factors of 12.
And also -1,-2,-3,-4,-6 and -12, because you get a positive number when you multiply two negatives, such as $(-2)\times(-6) = 12$
Answer: 1, 2, 3, 4, 6, 12, -1, -2, -3, -4, -6, -12
Factors are usually positive or negative whole numbers (no fractions), so $\frac{1}{2} \times 24 = 12$ is not listed.
Note: Negative numbers are also included, as multiplying two negatives makes a positive.

Example 4.

All the factors of 20.
Start at 1: 1×20=20, so put 1 at the start, and put its "partner" 20 at the other end:

1		20

Then go to 2. 2×10=20, so put in 2 and 10:

1	2		10	20

Then go to 3. 3 doesn't work (3×6=18, 3×7=21).
Then on to 4. 4×5=20, so put them in:

1	2	4		5	10	20

There is no whole number between 4 and 5 so you are done! (Don't forget the negative ones).

1	2	4	5	10	20
-1	-2	-4	-5	-10	-20

1.2 Squares and square roots of perfect squares

Squares of numbers

What you had learnt in the previous grade about multiplication will be used in this, to describe special products known as squares and square root.
The process of multiplying a number by itself is called **squaring** the number.

If the number to be multiplied by itself is 'a', then the product (or the result $a \times a$) is usually written as a^2 and is read as:

- ☑ a squared or
- ☑ the square of a or
- ☑ a to the power of 2

In geometry, for example you have studied that the area of a square of side length 'a' is $a \times a$ or briefly a^2.
The **square** of a number is the number multiplied by itself. The square of a number can be written as the number to the power of two.

Example 5.

The square of 5 is $5^2 = 25$

A **perfect square** is a non-zero whole number that is produced by multiplying a whole number by itself.

Activity 1

In pairs, find the square of each. The first pair to finish is the winner.

a) 8 b) 10 c) 14 d) 19

In pairs, solve and explain to the class how you did it.

a) 30^2 b) 40^2 c) 52^2

Example 6.

Find the square of 16.

This means 16 multiplied by itself.

$= 16\times16$

$= 256.$

256 is therefore the square of 16

A square can also be expressed as A^2

'A' being the number you want to square.

$25^2 = 25\times25$

$= 625$

Or $(25)^2 = 25x25$

$= 625.$

A number whose square root is exact is called a **perfect square.**

Activity 2:

1. with your partner choose a number. Write all the factors of that number. Now write what you know about factors. Share with the rest of the class

1. Find the squares of these numbers.

a. 21	b. 453	c. 17	d. 27	e. 19
f. 221	g. 305	h. 41	i. 34	j. 635

2. Find the value of;

a. 23^2	b. 18^2	c. 51^2	d. 32^2	e. 65^2
f. 39^2	g. 47^2	h. 33^2	i. 36^2	j. 36^2

Exercise 1.

1. A square has a side of 156cm. what is the area in square centimetres?
2. A fisherman has a pond of a square in shape. If one side of the pond was 126M. What was the area covered by the fish pond.
3. What is the area in square metres of a house whose one side is 27m?
4. Find the product obtained after working out the square of 13 and 15.
5. Rita planted flowers on a square garden of 31M. What is the area planted with flowers?

Square roots of numbers (Perfect squares)

The **square root** of a positive number is the number when multiplied by itself, produce the given number. The notation for square root is '$\sqrt{\ }$'.

For example, $\sqrt{25} = 5$

Activity 3

In pairs, find the square root of each of the following.

a) 100 b) 125 c) 169 d) 256 e) 625

Example 7.

$\sqrt{81}$

Methods 1: Using prime factorization method

Find $\sqrt{81}$

81

Divide 81 by 3 since it is not divisible by 2 3 27

3 9

Prime factors of 81 $= 3\times3\times3\times3$ 3 3

For every two same numbers pick one, and then find their product.

$$3\times3 = 9$$

$$\therefore \sqrt{81} = 9$$

Methods 2: Using average method.

Find $\sqrt{484}$

$484 \div 20$

$= \frac{484}{20}$

$= 24$ rem 4

$\frac{44}{2} = 22$

$\therefore$ Square root of 484 is 22

With the guide of the teacher; Estimate the square root of 484 to a round figure. Thus 20 is our estimated square root $20 \times 20 = 400$ which is closer to 484.

Divide the estimated square root by the number given. Where we get 24 remainder 4, ignore the remainder then add the divisor 20 to the quotient 24.

Then find the average of the result.

$20 + 24 = 44$ Divide by 2 to give 22.

To check whether 22 is our correct answer, multiply $22 \times 22 = 484$.

Exercise 2.

1. Fill in the correct missing numbers.

a. $\sqrt{144} = 12 \times$___ b. $\sqrt{169} =$ ___$\times 13$ c. $\sqrt{225} = 25 \times$___

d. $\sqrt{196} = 14 \times$___ e. $\sqrt{289} =$ ____$\times 17$

2. Use any of the two methods to find the square root of;

a. 441 b. 576 c. 1296 d. 2209

e. 5041 f. 2025

Show your working out.

Activity 4:

In groups discuss and then work out the following; share your workings using mathematical language.

1. A square has an area of 2 116cm^2. What is the perimeter in metres?
2. A farmer planted vegetables on a square area of 2401 square metres. What was the measurement of one side of the vegetable garden?
3. Garang had 5 square shaped cowsheds whose total area was 55125m^2. He wanted to fence it using 4 strands of wire. How many kilometers of wire was enough to fence the cowshed?
4. What is the square root of the number obtained when 2704 is divided by 16?
5. What is the value of $\sqrt{195 + 17^2}$?
6. What is the sum of the square of 19 and square root of 1225?
7. Find the difference between the square root of 3136 and 6084.
8. What is the value of the square of 23 multiplied by the square root of 676?
9. Find the value of the square root of 9216 and the square root of 256.
10. What is the square root of the number obtained when 196 is multiplied by 4?
11. What is the square root of the number obtained after 4498 is added to 263?
12. Find the sum of $\sqrt{1\frac{11}{25}} + 5\frac{1}{2}$

1.3 Squares and square roots of decimals and fractions

Squares of fractions

To square a fraction, you multiply the fraction by itself.

A fraction can also be squared by squaring the numerator and then squaring the denominator, as shown below.

Example 8.

$$\left(\frac{4}{7}\right)^2 = \frac{4}{7} \times \frac{4}{7} = \frac{4^2}{7^2} = \frac{16}{49}$$

To square a mixed number, write the mixed number as an improper fraction and then square the fraction.

Write the square of $\left(1\frac{4}{7}\right)^2$

Write the mixed fraction as an improper fraction. $=\left(\frac{11}{7}\right)^2$

Square the numerator and square the denominator separately. $=\frac{11^2}{7^2}$

$=\frac{121}{39}$

Write the result as a mixed fraction. $=2\frac{23}{49}$

Squares of decimal numbers

To find the squares of decimal numbers, change the given decimals into fractions with the denominator of 10.
E.g. 10, 100, 1000, 10 000, 100 000 and so on.
If the Fraction is a mixed fraction, we convert it to an improper fraction.

Example 9.

Evaluate 0.4^2

$= 0.4\times0.4 \ or \ \left(\frac{4}{10}\right)^2$

$=\frac{4}{10}\times\frac{4}{10}$

$=\frac{16}{100}$

$=\frac{16}{100}=0.16$

$=0.16.$

Exercise 3.

1. Work out:

a) 0.5^2 b) 0.03^2 c) 0.035^2 d) 3.06^2

e) 0.16^2 f) 1.8^2 g) 0.25^2 h) 0.075^2

i) 0.15^2 j) 0.27^2 k) 4.5^2 l) 3.87^2

m) 0.23^2 n) 0.033^2 p) 0.025^2

Square roots of fractions

The square root of a fraction can be obtained by finding the square root of the numerator and the square root of the denominator separately, as shown below.

Example 10.

$$\sqrt{\frac{81}{169}} = \frac{\sqrt{81}}{\sqrt{169}} = \frac{9}{13}$$

Square roots of decimals

To find the square root of decimals, write the given decimal number as a fraction with denominator of power 100.
E.g. 100, 10000, 1000000 and so on.
If the Fraction is a mixed fraction we convert it to an improper fraction.

Example 11.

(a) $\sqrt{0.36}$ (b) $\sqrt{0.0196}$ (c) $\sqrt{1.44}$

Solution

Find the square roots of both the numerator and the denominator.

(a) $\sqrt{0.36} = \frac{\sqrt{36}}{\sqrt{100}} = \frac{6}{10} = 0.6$

(b) $\sqrt{0.0196} = \frac{\sqrt{196}}{\sqrt{10000}} = \frac{14}{100} = 0.14$

(c) $\sqrt{1.44} = \frac{\sqrt{144}}{\sqrt{100}} = \frac{12}{10} = 1.2$

What do you notice about the decimal place of the given number and their square roots?

Numerators are significant figures of the decimal numbers.

Exercise 4.

Find the square root of:

a) 6.25 b) 2.25 c) 0.0169 d) 0.0144

e) 3.24 f) 26.01 g) 12.96 h) 3.61

i) 0.0081 j) 0.3136 k) 0.5625 l) 0.1225

1.4 Cubes of numbers

The cube of a number is the number raised to the power 3.

How do you find the volume of a cube of side s?

You multiply s by itself three times. Thus Volume of cube = $s \times s \times s = s^3$

When a number is multiplied by itself three times, we get the cube of the number.

The cube of 2 = $2^3 = 2 \times 2 \times 2 = 8$. The cube of 5 = $5^3 = 5 \times 5 \times 5 = 125$

8 and 27 are natural numbers which are cubes of natural numbers 2 and 3 respectively.

Such numbers are called cubic numbers or perfect cubes.

A cubic number or perfect cube is a natural number which is the cube of some natural number.

The following numbers are perfect cubes.

8($=2^3$) 27 ($=3^3$) 64 ($=4^3$) 125 ($=5^3$)

216 ($=6^3$) 343($=7^3$)

Exercise 5:

1. In pairs, find the cube of each number.

a. 4 b. 9 c. 16 d. 14

e. 30 f. 20

1.5 Ratios and proportions using the unitary method

The Unitary Method sounds like it might be complicated but it's not.
It's a very useful way to solve problems involving ratio and proportion.

Example 12.

If 12 tins of paint weigh 30kg, how much will 5 tins weigh?

Solution

The first step in solving this is to find what ONE tin weighs.

This will be $\frac{30}{12}$ so 2.5kg.

Then we scale this back up for 5 tins gives $5 \times 2.5 = 12.5$kg.

Activity 5

In groups find out the following;

1. If sixteen bricks weigh 192kg. What would nineteen bricks weigh?
2. If thirteen girls can plant 169 trees in a day. How many trees could fourteen girls plant in a day?

Explain your answers

Exercise 6:

For each question show your working out.

1. If twenty two workers can dig 308 holes in an hour. How many holes could twenty seven workers dig in an hour?
2. If thirty four coins weigh 170g. What would fifty one coins weigh?
3. If fifteen buses can seat 420 people. How many people could thirty five buses seat?
4. Thirty three identical pipes laid end to end make a length of 462m. What length would fifty seven pipes make if they are laid end to end?
5. 31 toy building blocks placed one on top of another reach a height of 341cm. How high would 79 blocks be if placed one on top of the other?
6. 960g of flour is needed to make a special cake for 16 people. How much flour would be needed to make a cake for 33 people?
7. A vehicle travels one hundred and ninety eight km on 18 litres of fuel. How far would it travel on twenty eight litres?
8. Another vehicle travels four hundred and sixty eight km on 39 litres of fuel. How far would it travel on seventy nine litres?

1.6 Percentage increase and decrease

The term 'per cent' means one out of a hundred.
In mathematics we use percentages to describe parts of a whole
The whole being made up of a hundred equal parts.
The percentage symbol % is used commonly to show that the number is a percentage.

To calculate the percentage increase:

First: *work out the difference (increase) between the two numbers you are comparing.*

Increase = New Number - Original Number

Then: *divide the increase by the original number and multiply the answer by 100.*

% increase = Increase ÷ Original Number × 100.

If your answer is a negative number then this is a percentage decrease.

To calculate percentage decrease:

First: *work out the difference (decrease) between the two numbers you are comparing.*

Decrease = Original Number - New Number

Then: *divide the decrease by the original number and multiply the answer by 100.*

% Decrease = Decrease ÷ Original Number × 100

You can also put the values into this formula:

$$PERCENT\ INCREASE = \frac{(new\ amount - original\ amount)}{original\ amount} \times 100\%$$

Example 13.

1. There were 200 customers yesterday, and 240 today:

$$\frac{240 - 200}{200} \times 100\% = \frac{40}{200} \times 100\% = 20\%$$

Answer A 20% increase.

2. But if there were 240 customers yesterday, and 200 today we would get:

$$\frac{200 - 240}{240} \times 100\% = \frac{-40}{240} \times 100\% = -16.6...\%$$

A 16.6...% decrease.

Exercise 7:

1. A price rose from SSP50000 to SSP70000. What percent increase is this?
2. A quantity decreased from SSP90000 to SSP75000. What percent decrease is this?
3. An item went on sale for SSP13000 from SSP16000. Write what you notice.
4. In a small town in South Sudan, the population has been slowly declining. In 2016 there were 2087 residents, and there were only 1560 residents in 2017. Work out the percent decline of the population.

5. Trees in our school increased from 90 trees to 120treees. What does this tell us about our school?

1.7 Conversion of fractions to percentage and percentage to fractions

Conversion of fractions to percentage

Percentage means out of hundred $\left(\frac{x}{100}\right)\%$

Example 14.

Express $\frac{3}{8}$ as a percentage.

$\frac{3}{8}$ Out of hundred

$= \frac{3}{\cancel{8}_{2}} \times \cancel{100}^{25}\% = \frac{75}{2}$

$= 35\frac{1}{2}\%$

Exercise 8.

1. Convert these fractions to percentages. Show your working out.

a) $\frac{3}{3}$ b) $\frac{1}{3}$ c) $\frac{5}{6}$ d) $\frac{13}{20}$ e) $\frac{5}{8}$

2. Johana scored 13 out of 18 in Kiswahili test. What were his marks as percentage? Show your working out.
3. In a class of 45 pupils there are 18 girls. What percentage of the total number of pupils were boys? Show your working out.
4. Pamela had 60 hens. She sold 15 hens. What percentages of hens were unsold? Show your working out.
5. A basket had 36 fruits, 27 of them were ripe. What percentage of fruits was raw? Show your working out.

6. Abdi had 600 cattle. If he had 180 dairy cattle. What percentage were beef cattle?

7. In a tray there were 30 eggs. 11 eggs were rotten. What percentages of eggs were good?

Activity 6:

In groups discuss, where do we apply converting fractions to percentages.

Conversion of percentage to fractions

Example 15.

Write 35% as a fraction and write in simplest form.

35% is 35 out of 100

Change to a fraction $= \frac{35}{100}$

Simplify by cancelling the numerator and the denominator by a common divisor. $\frac{\cancel{35}}{\cancel{100}} = \frac{7}{20}$

$= \frac{7}{20}$

Example 16.

Express $33\frac{1}{3}\%$ as a fraction.

$\frac{33\frac{1}{3}\times 3}{100\times 3} = \frac{100}{300}$

$= \frac{\cancel{100}}{\cancel{300}} = \frac{1}{3}$

Multiply numerator and denominator by 3 to get whole number to get $\frac{100}{3\times 100} = \frac{100}{300}$

Then simplify by dividing 300 by 100 to get $\frac{1}{3}$

Exercise 9.

Convert these percentages to fractions in the simplest form.

a) 60% b) 75% c) 90% d) $32\frac{1}{2}\%$

e) $27\frac{1}{2}\%$ f) $2\frac{1}{4}\%$ g) $37\frac{1}{2}\%$ h) $66\frac{2}{3}\%$

Chose two questions tell your partner how you converted this percentages to fractions.

1.8 Conversion of decimals to percentage and percentage to decimals

Conversion of decimals to percentages

Example 17.

Express 0.05 as percentage.

Change to a fraction and multiply by 100%

$$0.05 = \frac{5}{\cancel{100}} \times \cancel{100}$$

$$= 5\%$$

Change it into fraction first i.e. $\frac{5}{100}$.

Then multiply by 100 and cancel 100 by 100 to get 5%

Activity 7:

In pairs, express the following as percentage.

a) 0.567 b) 0.4 c) 0.036 d) 0.48 e) 0.135

f) 1.75 g) 0.23h) 2.8 i) 0.25 j) 3.75

Conversion of percentages to decimals

Example 18.

Convert 88% to a decimal

$88\% = \frac{88}{100} = 0.88$

Change it into fraction.

Then divide by 100

$= 0.88$

Exercise 10.

Convert the following percentages to decimals.

a) 77%	b) 135%	c) 265%	d) 1%
e) 857%	f) 13%	g) 175%	h) 8%
i) 19%	j) 9%		

Activity 8:

Where do we apply converting fractions to percentages?

1.9 Application of fractions, decimals and percentage

When we talk, we often use different words to express the same thing. For example, we could describe the same car as **tiny** or **little** or **small.** All of these words mean the car is not big.
Fractions, decimals, and percents are like the words tiny, little, and small. They're all just different ways of expressing parts of a whole.

Fractions

Fractions are used in the real world during jobs such as a chef or a baker because you need to know how much of something like butter or milk to put in a recipe

Decimals

Decimals are used in measurements for example my pen is 5.5 inches long. Architects use decimals when they are measuring the height of a building.

Percent

In restaurant they have to use percent when they make a pizza so that they can cut it into equal pieces. Finally they use them when they decide how much of their budget goes to supplies.

Example 1.9

In real life, fractions are used in games like soccer we talk of half time, as they are splint in to halves. Also fractions are used in food, i.e. $\frac{1}{2}$ cup of sugar.

In real life, percentages are used in liquids and food. i.e. 30% of tea is milk, 100% percent orange juice. Also in washing, we say what percentage of germs will be killed and how safe it is. i.e. 100% safe and kills 99.9% germs

Exercise 11.

In pairs, work out the following and share your working with your partner.

1. In a closing-down sale a shop offers 50% off the original prices. What fraction is taken off the prices?
2. In a survey one in five people said they preferred milk. What is this figure as a percentage?
3. Mary is working out a problem involving $\frac{1}{4}$. She needs to enter this into a calculator. How would she enter $\frac{1}{4}$ as a decimal on the calculator?

4. Deng pays tax at the rate of 25% of his income. What fraction of Deng's income is this?
5. When a carpenter was buying his timber, he had to put down a deposit of $\frac{1}{10}$ the value of timber. What percentage was this?
6. I bought my coat in January with $\frac{1}{3}$ off the original price. What percentage was taken off the price of the coat?
7. Brian bought a cloth that was 1.75 metres long. How could this be written as a fraction?

UNIT 2: MEASUREMENT

2.1 Perimeter of rectangle, square, triangle, circle and trapezium

Perimeter is the distance around a shape. Its symbol is **P.** In order to calculate the perimeter of a shape, you must add up the lengths of all its sides.

Example 1.

A rectangle has a width of 5cm and a length of 3cm, its perimeter would be:

3cm

5cm

PERIMETER = SUM OF LENGTH OF ALL FOUR SIDES

$= 5 + 3 + 5 + 3$

$= 16$cm

There are different types of geometric shapes.

They include:
Rectangle, Square, Triangle, Circle, Trapezium
There is a formula for calculating the perimeter of each shape.

Perimeter of a rectangle

A Rectangle is a four sided with two opposite sides equal to each other. The longer side is called the **Length** while the shorter side is called the **Width.**

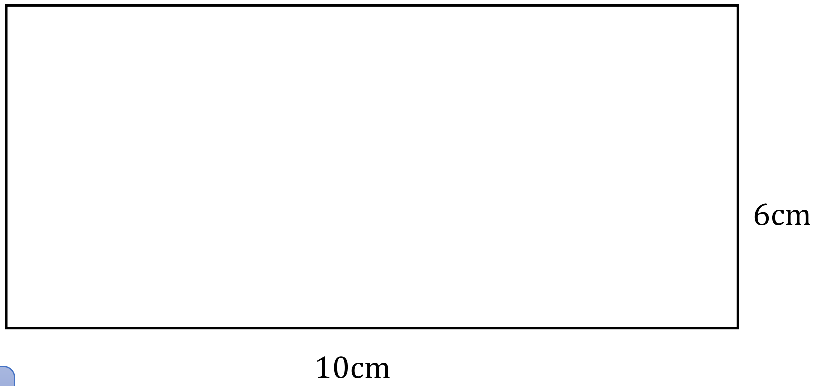
Perimeter of a rectangle = 2 (Length + Width)

$P = 2(L+W)$

$P = 2(10+6)$

$P = 2 \times 16$

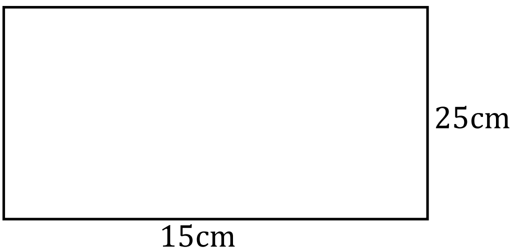
$P = 32$ cm



Exercise 1.

1. Find the perimeter of the following:

1.



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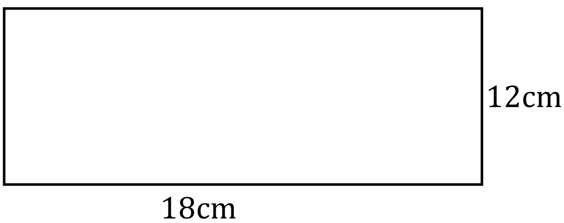

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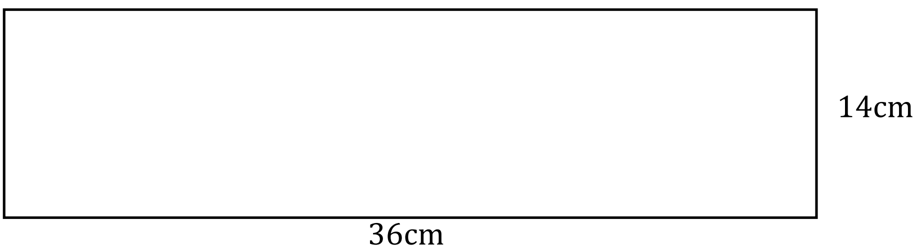


4. A rugby makes seven runs around a rugby field of length 90m and width 75m. Calculate the distance she covered.
5. A farmer wants to fence a field of length 800m and width 650m by surrounding it with a barbed wire, calculate the length of the barbed wire used.
6. To fence a rectangular plot of length 150m and width 100m, a landlord erects poles which are 50m apart. How many poles are required?

Activity 1:

Work in groups;

1. The perimeter of a rectangular playground is 46 m. If the length of the park is 7 m, what is the width of the park? Explain your working.
2. The perimeter of a rectangular field is 60 M and its width is 20 M. Find the perimeter of this field. Show your workings.
3. Before soccer practice, Laura warms up by jogging around the soccer field that is 80 M by 120 M. How many yards does she jog if she goes around the field two times?

<u>Perimeter of a square</u>

A Square is a four sided with all sides equal to each other.

Perimeter of a Square = 4 × Length

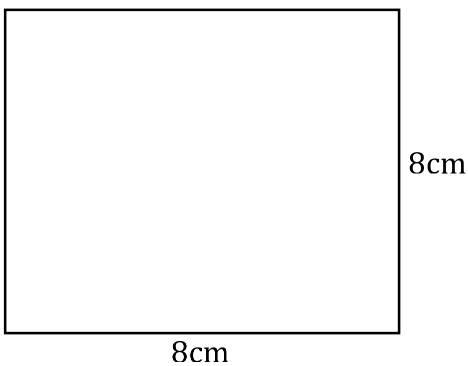
Example 2.

P = 4L

 = 8cm

P = 4 X 8cm

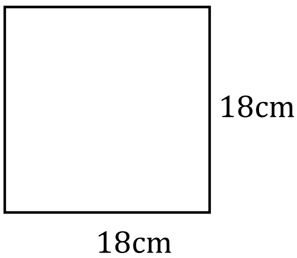
P = 32 cm



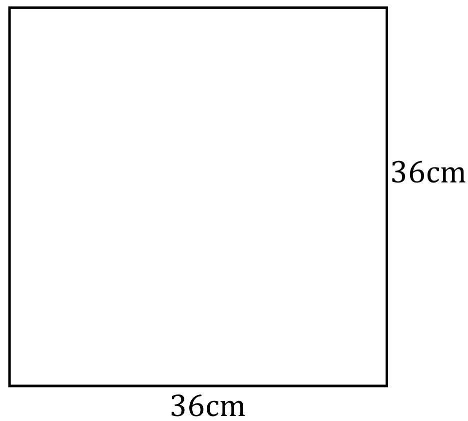
Exercise 2.

1. In groups, find the perimeter of the following:

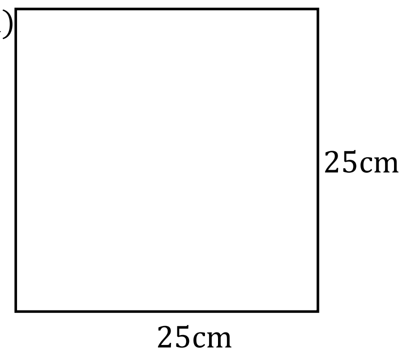
(i)



(iii)



(ii)



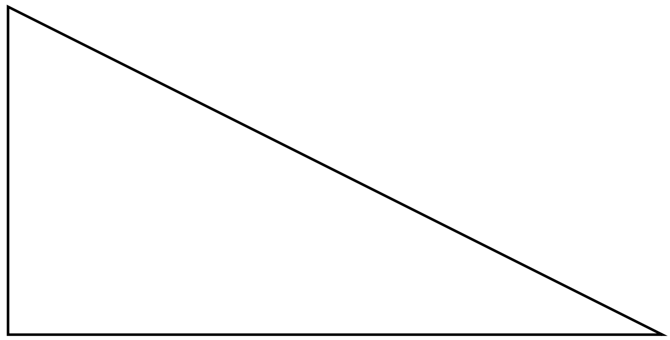
Activity 2:

Work in groups;

1. A cricket player makes four runs around a pitch of length 90m. Calculate the distance he covered. Explain how you arrived at your answer.
2. A farmer wants to fence a square field of length 800m by surrounding it with a barbed wire, calculate the length of the barbed wire used. Present your working.

Perimeter of a triangle

A triangle is a three sided figure.



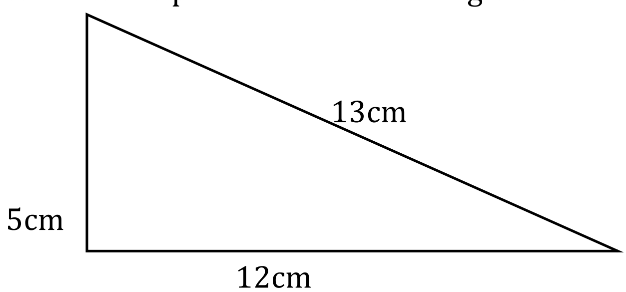
Perimeter of triangle is the sum of the lengths of all the sides.
Like any polygon, the perimeter is the total distance around the outside, which can be found by adding together the length of each side.
Or as a formula:

$$Perimeter = a + b + c$$

Where: a, b and c are the lengths of each side of the triangle.

Example 3.

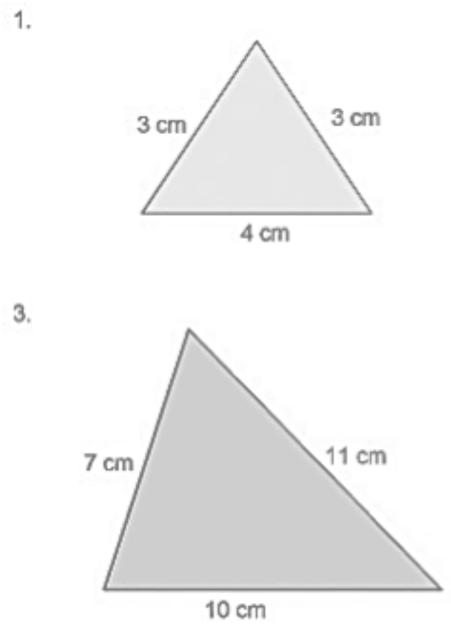
Determine the perimeter of the triangle below:



Perimeter $= \text{Sum of length of sides}$
$= 12 + 13 + 5$
$= 30\text{cm}$

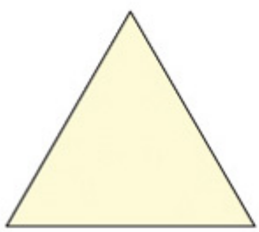
Exercise 3.

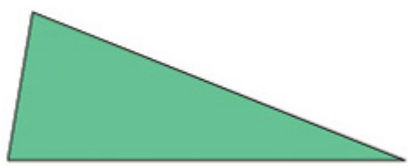
In pairs, work out the perimeters of the triangles below. (Not drawn to scale)



Activity 3:

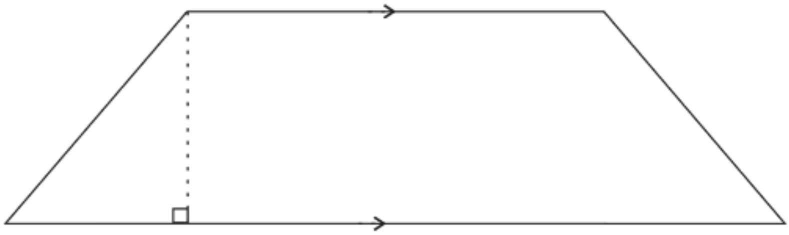
Measure the lengths of the sides of this triangles and calculate their perimeters.





Perimeter of a trapezium

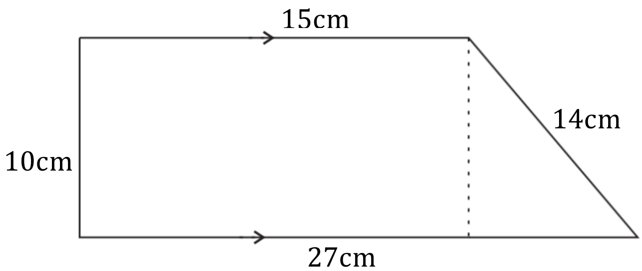
A trapezium has two parallel sides with one of the sides being shorter than the other.



The perimeter of a trapezium is the sum of the distances round the figure.

Example 4.

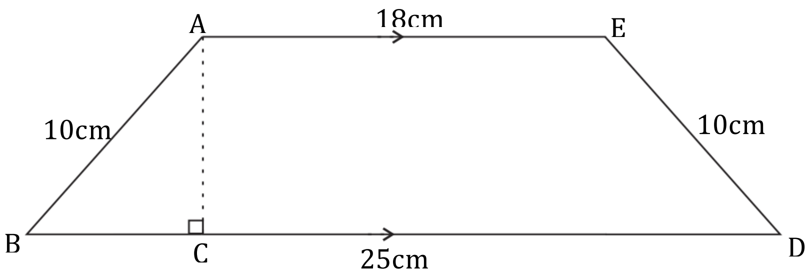
Calculate the perimeter of the figure below:



P $= 15 + 27 + 14 + 10$
$= 66$ cm

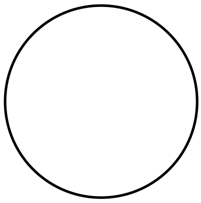
Activity 4:

Find the perimeter of the following figures:



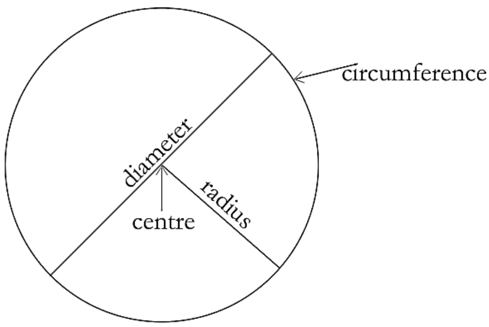
Perimeter of a circle

The distance round a circle is known as the circumference. The symbol for circumference is **C**.



A circle has a line joining two points of the circle which cuts through the centre. The line is known as a **diameter (d)**. The distance between the centre of a circle and any point on the circumference is called a **radius (r)**.
The distance from the circumference through the centre is its **diameter (d)**

The distance from the centre to the circumference is its **radius.**
2 radii (plural for radius) = diameter.



Activity 5

Using a pair of compass draw a suitable circle on a manilla paper.

- Use a pair of scissors blade to cut it carefully around its circumference.
- Using a string measure the circumference.
- Use the circumference to find out how many times it can fit on the diameter.
- It will fit approximately $3\frac{1}{7}$ or 3.14 times around the diameter.
- Circumference of a circle divided by its diameter is approximatly $3\frac{1}{7}$ or 3.14

- $3\frac{1}{7}$ or $\frac{22}{7}$ or 3.14 are used as appriximations for π read as pi.

$$C \div d = \frac{C}{d} = \pi$$

$$C = \pi d$$

Or $\quad C = 2\pi r$

In groups, peg a string on the ground to form center of a circle and draw a circle. Measure the line you have drawn.

Pi (π) is approximately $\frac{22}{7}$ or 3.14.

Example 5.

Approximate the circumference of a circle with diameter of 7cm.

Circumference $= \pi d$

$= \frac{22}{7} \times 7\text{cm}$

$= 22\text{cm}$

Circumference can also be calculated using radius.

Circumference $= 2\pi r$

C $= 2\,\pi r$

Example 6.

Find the circumference of a circle with radius of 7cm.

Circumference $= 2\pi r$

C $= 2\,\pi r$

C $= 2\times\frac{22}{7}\times 7\text{cm}$

C $= 44\text{cm}$

Exercise 4.

1) The radius of a circle is 5 inches. What is the

(a) Diameter (b) circumference

2) The diameter of a circular rug is 35 cm.

a) What is the radius? b) Circumference? c) Radius?

The radius, diameter, or circumference of a circle is given. Find the missing measures. Show your work.

Radius	Diameter	circumference
10inch		
	10	
		76.4
10.4		
		314
	1	

Activity 6:

1. In pairs, find the following;
   a) The circumference of a circle with a diameter of 15 *cm*.
   b) The radius of a circle with a diameter of 480 *in*.
   c) The circumference of a circle with a radius of 320 *cm*.
   d) The diameter of a circle with a radius of 522 ft.
2. The diameter of a bicycle wheel is 34 inches. What is the radius?
   a. How far will you move in one turn of your wheel?
   b. What is the distance covered in 5 turns of the wheel? Show your work to your partner.

Semi-circle

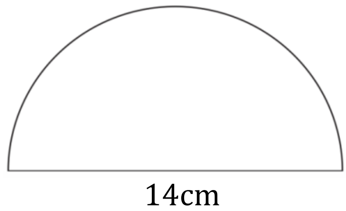
A half of a circle is called a semi-circle.
The circumference of a semi-circle is:

Circumference $=\frac{1}{2}\pi d + d$

Divide the formula by 2 or multiply by $\frac{1}{2}$ because it is $\frac{1}{2}$ of a circle then add the diameter to get the distance all the way round.

Example 7.

Determine the circumference of the figure below:



C $= \frac{1}{2}\pi d + d$

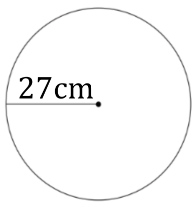
$= (\frac{1}{2} \times \frac{22}{7} \times 14) + 14$

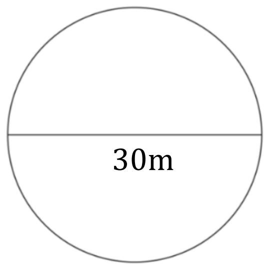
$= 36\text{cm}$

Exercise 5.

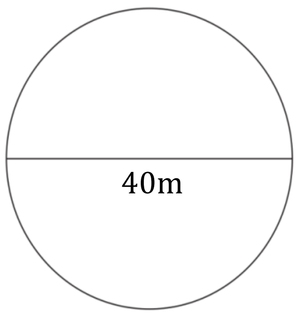
1. Determine the circumference of the following figures:

a.



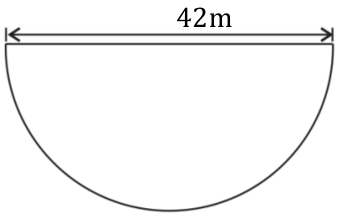


b.



2. A motorcyclist is racing round a circular course of radius 49m. Determine the distance he makes five runs round the course.
3. Determine the circumference of a semicircular field of diameter 18 metres.
4. Calculate the diameter and radius of a circle whose circumference is 77cm.

5. To fence a circular field of diameter 35m, a farmer erects poles after every 10m. Determine the number of poles the farmer requires to fence the field.
6. A plot of land is in the shape of a semi-circle of diameter 42m as shown below.



The plot was fenced by erecting posts 3m apart. How many posts were used?

Activity 7:

Working in groups, make a fact sheet to help learners understand how to work out the perimeter of two shapes

2.2 Area of rectangle, square, triangle, circle and trapezium

The area is the amount of a surface covered by a boundary. The symbol for area is **A.**

The units for area are square units such as square metres (m^2), square centimeters (cm^2), square kilometers (km^2), Ares, hectares (Ha).

$1m^2 = 1m \times 1m$

$1m^2 = 10000cm^2$

$1 \text{ Are} = 100m^2$

$1\text{Hectare} = 10000\ m^2$

Example 8.

To convert m^2 to cm^2, multiply the value given by 10000.

1. Convert $1.8m^2$ to cm^2

<u>**Solution**</u>

$1m^2 = 10000cm^2$

$1.8m^2 = 1.8 \times 10000$

$= 18000cm^2$

2. Convert $0.075m^2$ to cm^2

Solution

$1m^2 \quad = 10000cm^2$

$0.075m^2 = 0.075 \times 10000$

$\quad\quad = 750cm^2$

To convert cm^2 to m^2, divide the value given by 10000.

1. Convert $1500cm^2$ to m^2

Solution

$10000cm^2 \quad = 1m^2$

$1500cm^2 \quad = \frac{1500}{10000}$

$\quad = 0.15m^2$

2. Convert $28450cm^2$ to m^2

Solution

$10000cm^2 \quad = 1m^2$

$28450cm^2 \quad = \frac{28450}{100000}$

$\quad = 2.845m^2$

Area of a rectangle

A Rectangle is a four sided with two opposite sides equal to each other. The longer side is called the **Length** while the shorter side is called the **Width.**

Area of a rectangle = Length x Width)

A = L × W)

A = 15 × 6

A = $90cm^2$

Activity 8:

In groups, solve the questions

1. The floor of a classroom has a length of 12m and a width of 9m. Calculate its area.
2. A farmer has a rectangular garden of length 800m and width 650m. Calculate the area of the garden on hectares.
3. A football field has length of 90m and a width of 75m. What is the area of its playing surface?

Area of a square

A Square is a four sided with all sides equal to each other.

Area of a Square = Length × Length

$A = L^2$

$A = 8^2$

$A = 64cm^2$

Activity 8:

1. Find the area of the following:

   (i) A square of sides 25cm.

   (ii) A square of sides 18cm.

   (iii) A square of sides 36cm.

3. A designer is using carpet to cover the floor of a room of area $169m^2$. Determine the dimensions of the carpet used.

Area of a right angled triangle

A triangle is a three sided figure.

Area of triangle $= \frac{1}{2} \times base \times height$

$$A = \frac{1}{2}bh$$

The height and the base are the two sides which form a rich angle. The longest side of the triangle is called the hypotenuse. It is not used in calculation of the area.

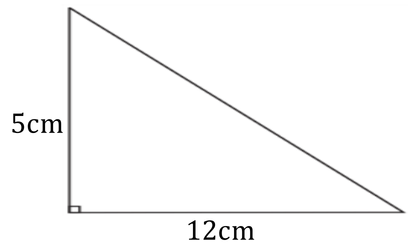
Example 9.

Determine the area of the triangle below:

Area $= \frac{1}{2}bh$

$= \frac{1}{2} \times 12 \times 5$

$= 30\text{cm}^2$



Exercise 6.

1. A triangle with an area of 150cm^2 has a base of 20cm. Calculate its height.
2. A triangular plot has a base of 12m and hypotenuse of 13m. Calculate its area.

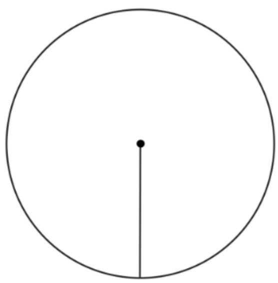
Area of a circle

A circle has a line joining two points of the circle which cuts through the centre. The line is known as a **diameter (D).**

The distance between the centre of a circle and any point on the circumference is called a **radius (r).**

Area = pi × (radius)^2

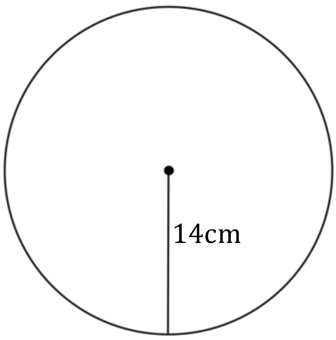
A $= \pi r^2$



Pi π *is apprixmately* $\frac{22}{7}, 3\frac{1}{7}$ *or* 3.14

Example 10.

Find the area of the circle below:



Area of a circle $= \pi \times r \times r$

$$\pi = \frac{22}{7}, 3\frac{1}{7} \ or \ 3.14$$

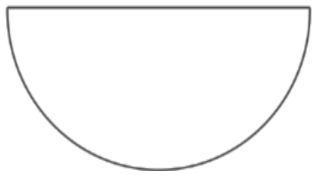
$$\text{Area} = \frac{22}{\cancel{7}} \times \overset{2}{\cancel{14}}cm \times 14cm$$

$$= 44 \times 14cm^2$$

$$= 616cm^2$$

Semi-circle

A half of a circle is called a semi-circle.



The area of a semi-circle is:

Circumference $= \frac{1}{2} \times pi \times (Radius)^2$

Example 11.

Determine the area of a semicircle of diameter 14cm.

A $= \frac{1}{2}\pi^2$

$= \frac{1}{2} \times \frac{22}{7} \times 7^2$

$= 77\text{cm}^2$

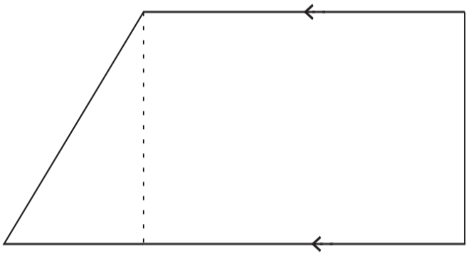
Exercise 7.

1. Determine the area of the following:
    a. Circle of radius 21 cm.
    b. Circle of diameter 30cm.
    c. Semi-circle of diameter 40cm.

2. A motorcyclist is racing round a circular course of radius 49m. Determine the area of the course.
3. A circular shaped lake covers a diameter of 7km. Determine the area of the surface covered by the lake.
4. Calculate the area of a circle which has a radius of 4cm
5. A circular playing field has an area of $242m^2$. Calculate its circumference.
6. A semicircular disk has a diameter of 7cm. Find its area in square metres.

Area of a trapezium

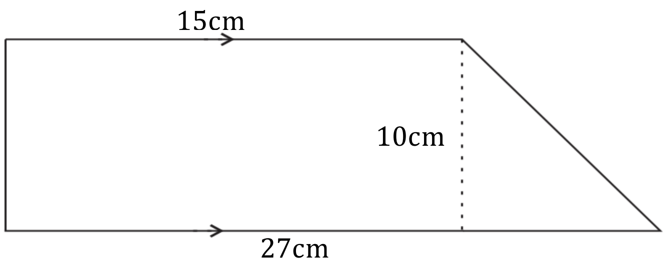
A trapezium has two parallel sides with one of the sides being shorter than the other. The two parallel sides are joined on one end by a height.



Area of a trapezium $=\frac{a+b}{2} \times height$.

Example 12.

Calculate the area of the figure below:

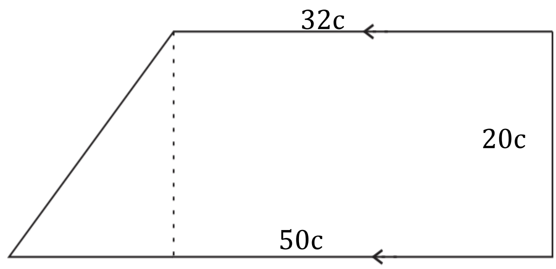


A $= (15 + 27)$ x 10

$= 210cm^2$

Activity 9:

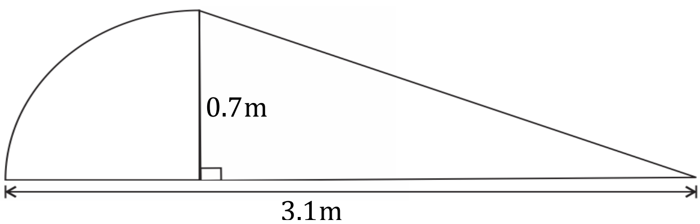
1. The two parallel sides of a trapezium shaped field are 280m and 160m. The field has an area of 3.3 hectares. Calculate the width of the field.
2. Calculate the area of figure below.



Exercise 8.

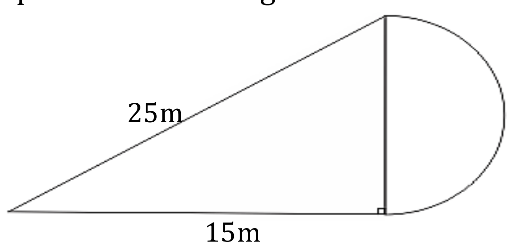
Show your working out. π ***is appriximately*** **3.14**

1. The figure below represents a swimming pool in the shape of a quarter of a circle of radius 0.7m and a right-angled triangle.

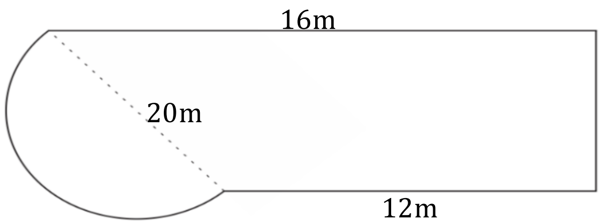


What is the area in cm^2 of the swimming pool?

2. The figure below represents a flower garden. What is the area in m^2?

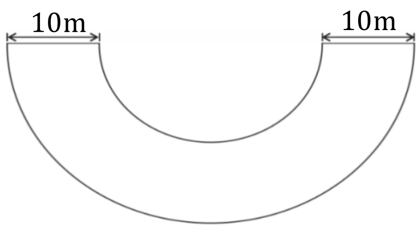


3. A cow shed is of the shape shown below, formed by a semi-circle and a trapezium.



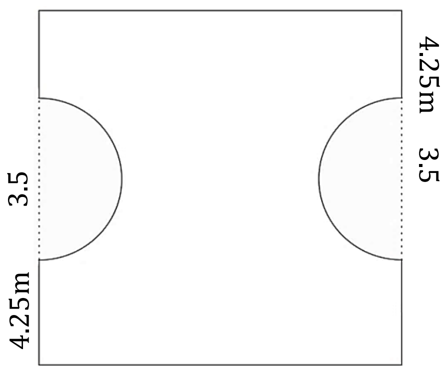
What is the area of the cow shed in square metres?

4. The figure below represents a potato garden enclosed by two semi circles 10m apart. The diameter of the larger circle is 40m.



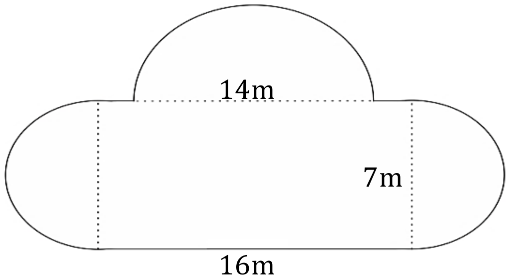
What is the area covered by the potato garden?

5. The figure below represents a flower garden formed by a square and two semi circles each of diameter 3.5m.

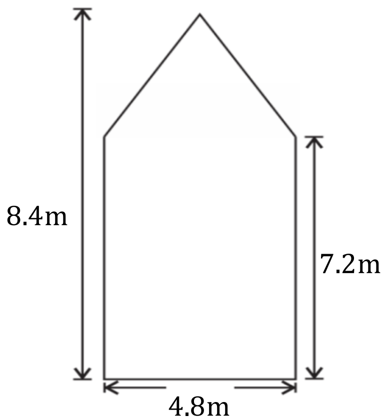


What is the area of the garden in square metres?

6. Find the area of the figure shown below?

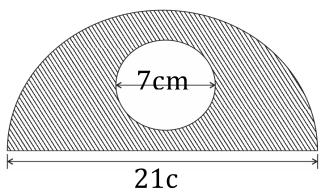


7. The diagram below represents a shape of a grazing field.

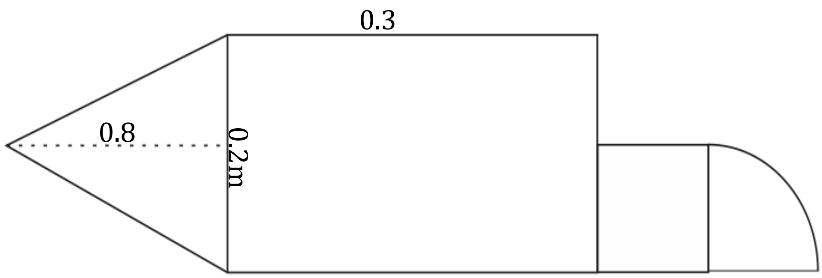


What was the area for grazing?

8. Find the area of the shaded figure below with a circle in the semi-circle.

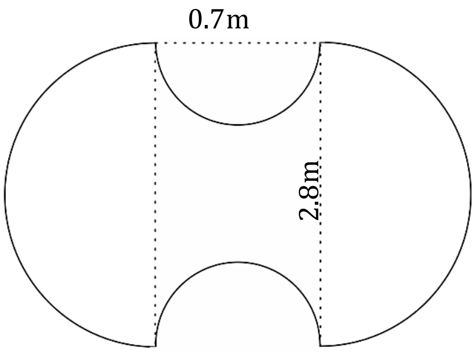


9. A piece of land is in the shape shown below. It consists of an isosceles triangle, a square and a quarter of a circle.



If the base of an isosceles triangle, is half one side of the square. What is the area of the whole figure in square centimetres?

10. The figure represents a piece of cardboard used to make certain furniture, with two opposite semi-circles.

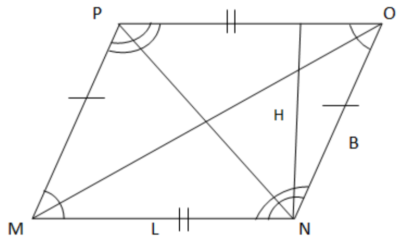


What is the area in square metres of the cardboard?

2.3 The Relationships between Quadrilaterals

There are many different types of quadrilaterals and they all share the similarity of having four sides, two diagonals and the sum of their interior angles is 360 degrees. They all have relationships to one another, but they are not all exactly alike and have different properties.

Parallelogram



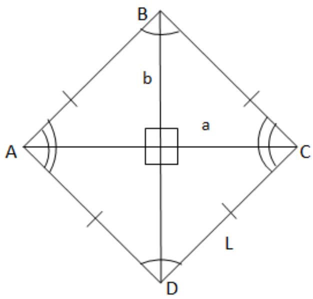
Properties of a parallelogram

- Opposite sides are parallel and equal.
- Opposite angles are equal.
- Adjacent angles are supplementary.
- Diagonals bisect each other and each diagonal divides the parallelogram into two equal triangles.

Important formulas of parallelograms

$$Area = L \times H$$

Rhombus



Properties of a Rhombus

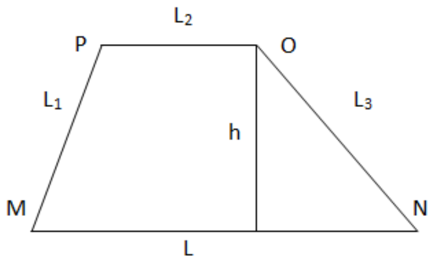
- All sides are equal.
- Opposite angles are equal.
- The diagonals are perpendicular to and bisect each other.
- Adjacent angles are supplementary (For eg., $\angle A + \angle B = 180°$).
- A rhombus is a parallelogram whose diagonals are perpendicular to each other.

Important formulas for a Rhombus

If a and b are the lengths of the diagonals of a rhombus,

$$Area = \left(\frac{a \times b}{2}\right)$$

Trapezium



Properties of a Trapezium

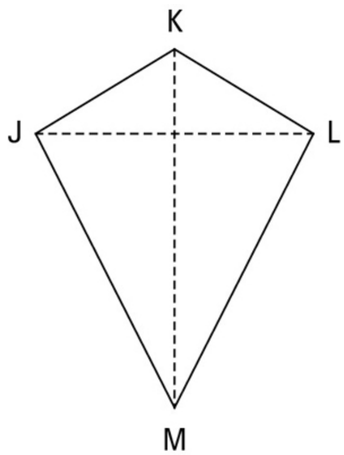
The bases of the trapezium are parallel to each other (MN ∥ OP).
No sides, angles and diagonals are equal.

Important Formulas for a Trapezium

$$Area = \left(\frac{1}{2}\right) h \left(L + L_2\right)$$

Kite

A kite is a quadrilateral in which two disjoint pairs of consecutive sides are congruent ("disjoint pairs" means that one side can't be used in both pairs). Check out the kite in the below figure.



The properties of the kite are as follows:

- ☑ Two disjoint pairs of consecutive sides are congruent by definition

  ($\overline{JK} \cong \overline{LK}$ and $\overline{JM} \cong \overline{LM}$).

  Note: *Disjoint* means that the two pairs are totally separate.

- ☑ The diagonals are perpendicular.

- ☑ One diagonal (segment *KM*, the *main diagonal*) is the perpendicular bisector of the other diagonal (segment *JL*, the *cross diagonal*). (The terms "main diagonal" and "cross diagonal" are made up for this example.)

- ☑ The main diagonal bisects a pair of opposite angles (angle *K* and angle *M*).

- ☑ The opposite angles at the endpoints of the cross diagonal are congruent (angle *J* and angle *L*).

Summary of properties

Summarizing what we have learnt so far for easy reference and remembrance:

Activity 10

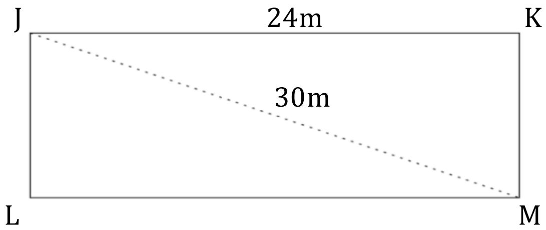
In groups, draw and cut out shapes of Parallelogram, Rhombus, square, rectangle and trapezium. List down different properties that can be observed from the shapes. Present them to the class using mathematical vocabulary, in a table

Activity 11

In groups, play the gues my shape game. (Instructions in the teachers guide)

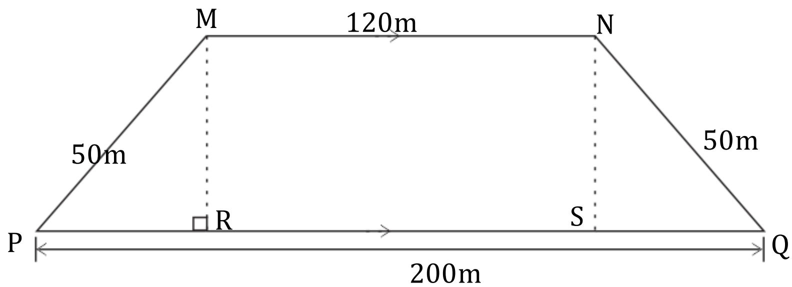
Exercise 9:

1. The figure below shows a rectangular grass lawn $JKLM$ in which $JK = 24m$ and $JM = 30m$.

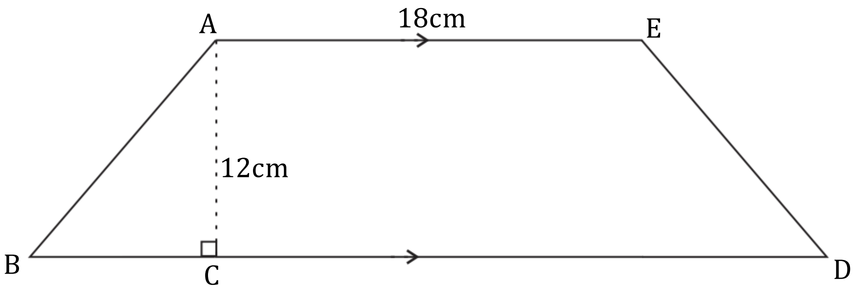


What is the area covered by grass in m^2?

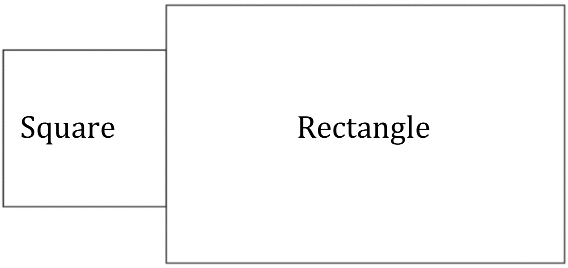
2. What is the area in hectares of the figure shown below? Where $PR = SQ$ and MN is parallel to PQ and $MP = NQ = 50m$ and angle $PRM = 90^0$



3. What is the area of a rhombus whose diagonals are 8m and 5m long in square metres?
4. The perimeter of a rectangular plot of land is 280 metres. The width is 60 metres. What is the area of the plot?
5. The diagram $ABCDE$ is a trapezium. If its area is $208cm^2$, what is the measure of CD in cm?



6. The diagram below shows the shape of Ruth's house which is formed by a square and a rectangle. The area of the square is 196cm². If the area of the square is $\frac{1}{4}$ that of the rectangle, what is the width of the rectangle in centimetres, if the length is 49cm? explain your method of working out this to your partner

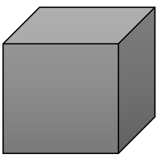


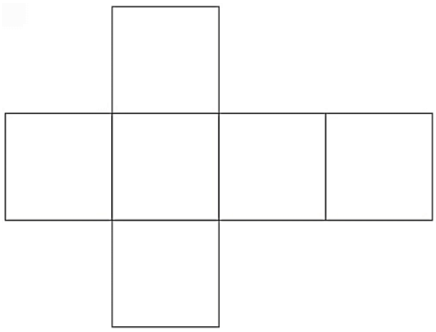
2. 4 area of a cube and cuboid

Surface area of a cube

A cube has 6 equal square faces.

An open cube has 5 equal square faces.





Formula of the surface area of a closed cube.

$$Surface\ area = (L \times W) \times 6\ faces$$

Example 13.

Find the surface area of a closed cube whose one side is 8m.

$$Surface\ area\ of\ a\ closed\ cube = (8 \times 8)m^2 \times 6$$

$$= 64m^2 \times 6 = 384m^2$$

Surface area of a cuboid

A cuboid has 2 equal opposite sides.

$$Surface\ area\ of\ a\ closed\ Cuboid\ = 2(L \times W) + 2(L \times W) + 2(L \times W)$$

Formula of the surface area of a closed cuboid.

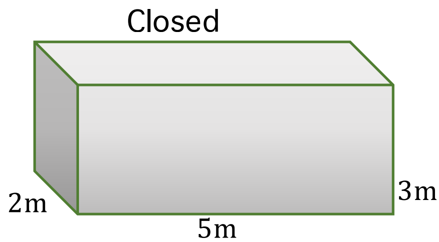
$$Surface\ area\ of\ a\ closed\ Cuboid$$
$$= 2(L \times W) + 2(L \times W) + 2(L \times W) square\ units$$

Activity 12:

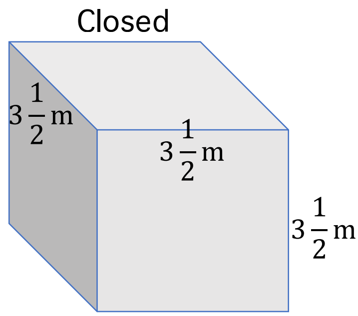
In groups, work out the questions below and explain your mathematical workings to other groups.

1. Find the surface area of the cubes and cuboids below.

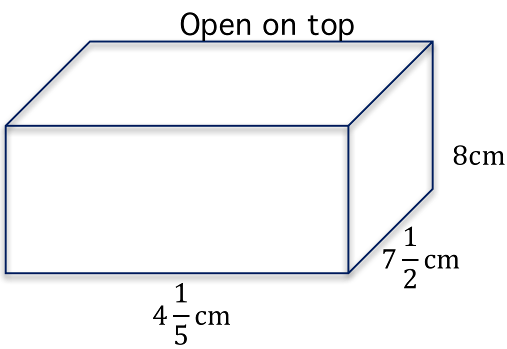
a)



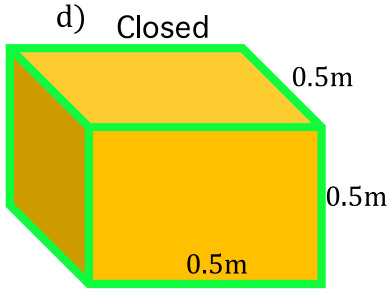
b)



c)



d)



2. One closed cuboid measures 4m by 1.5m by 2m, another open cuboid measures 8cm by 6cm by 5cm. What is the difference in their surface areas in square centimetres?

3. An open cuboid tin of 12cm by 9cm by 7cm was painted on the outside. What was the area painted altogether?

4. The base of a closed cuboid measures $4\frac{1}{2}$ cm by $5\frac{1}{2}$ cm by $7\frac{3}{4}$ cm. The base and the top part of the cuboid are not painted. What is the total surface area of the parts which are not painted?
5. The volume of an open rectangular tank is $48.6m^3$. The tank has a square base. The height of the tank is 5.4m. What is the surface area of the tank in square metres?

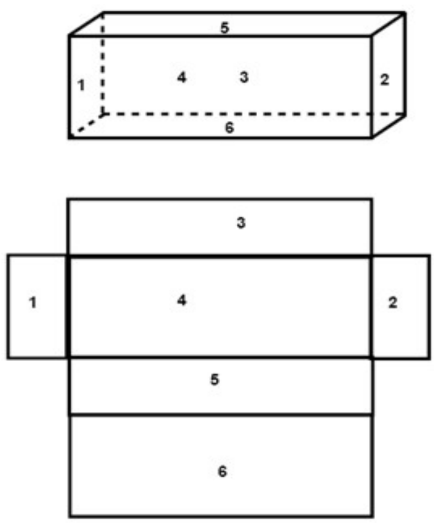
2.5 Surface area of common solids

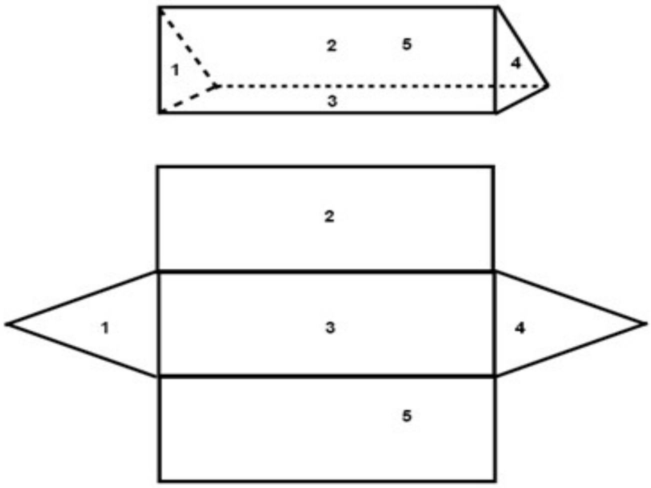
The surface area is the area that describes the material that will be used to cover the solid.
When we determine the surface areas of a solid we take the sum of the area for each geometric form within the solid.
The volume is a measure of how much a figure can hold and is measured in cubic units. The volume tells us something about the capacity of a figure.

Surface area of a prism

A prism is a solid that has two parallel congruent sides that are called bases that are connected by the lateral faces that are parallelograms. There are both rectangular and triangular prisms

To find the surface area of a prism (or any other geometric solid) we open the solid like a carton box and flatten it out to find all included geometric forms.



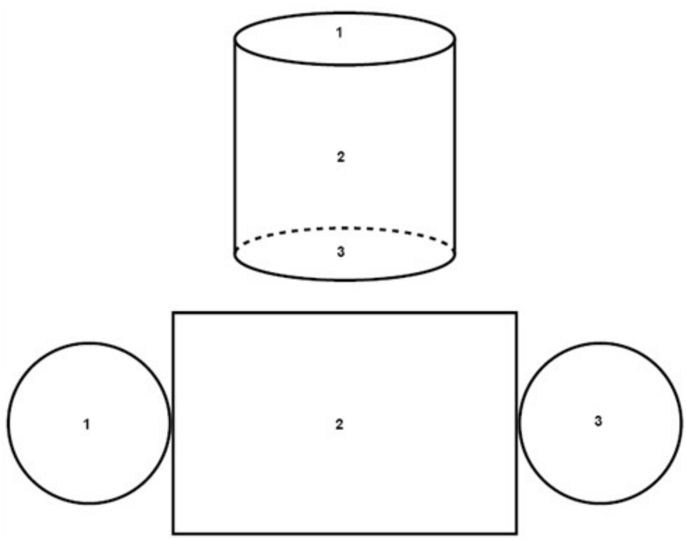


To find the volume of a prism (it doesn't matter if it is rectangular or triangular) we multiply the area of the base, called the base area B, by the height h.

$$V = Base\ area \times height = B \times h$$

Surface area of a cylinder

A cylinder is a tube and is composed of two parallel congruent circles and a rectangle which base is the circumference of the circle.



Example 14.

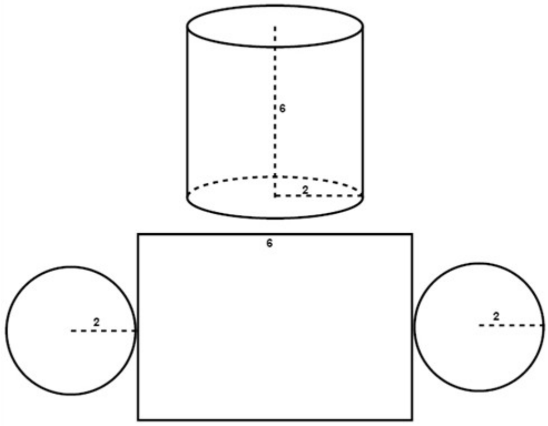
The area of one circle is:

$A = \pi r^2$

$A = \pi \times 2^2$

$A = 3.14 \times 4$

$A = 12.56$



The circumference of a circle:

$C = \pi d$

$C = 3.14 \times 4$

$C = 12.56$

The area of the rectangle:

A=Ch

A=12.56×6

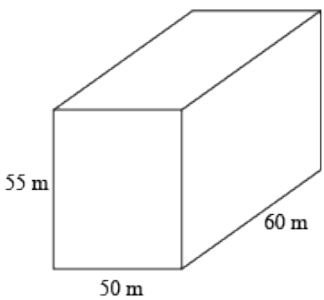
A=75.36

The surface area of the whole cylinder:

A=75.36+12.56+12.56=100.48units2

Exercise 10:

1. Find the total surface area of the solid.



2. Find the surface areas of the following..
   a. A cube of side length 1.5 m.
   b. A rectangular prism 6 m × 4 m × 2.1 m.
   c. A cylinder of radius 30 cm and height 45 cm, open at one end.
   d. A square pyramid of base length 20 cm and slant edge 30 cm.

2.6 Converting m^3 to cm^3

To convert m^3 to cm^3, multiply the value given by 1000000.

Example 15.

1. Convert $13.8m^3$ to cm^3

$$1m^3 = 1000000cm^3$$
$$13.8m^3 = 13.8 \times 1000000$$
$$= 13800000cm^3$$

2. Convert $0.075m^3$ to cm^3

$$1m^3 = 10000cm^3$$
$$0.075m^3 = 0.075 \times 1000000$$
$$= 75000cm^3$$

To convert cm^3 to m^3, divide the value given by 10000.

Example 16.

1. Convert $1500cm^3$ to m^3

$$1000000cm^3 = 1m^3$$
$$1500cm^3 = \frac{1500}{1000000}$$
$$= 0.0015m^3$$

2. Convert $28450cm^3$ to m^3

$$1000000cm^3 = 1m^3$$
$$28450cm^3 = \frac{28450}{1000000}$$
$$= 0.02845m^3$$

2.7 Volume of a cube and cuboid

Volume is the space occupied by matter. It can also be defined as the space enclosed by matter. The symbol for volume is **V**.

3.5.1 Volume of cubes and cuboids

$$Volume = Base\ area \times Height$$

Example 17.

Find the volume of the cube whose one side is 15Cm.

Solution

A cube is square based and all sides are equal.

$$\text{Base area} = L \times W$$
$$V = L \times W \times H$$
$$= (15 \times 15 \times 15)cm^3$$
$$= 3375cm^3$$

Volume is given in cubic units such as cubic centimetres (cm^3), cubic metres (m^3) etc.

Example 18.

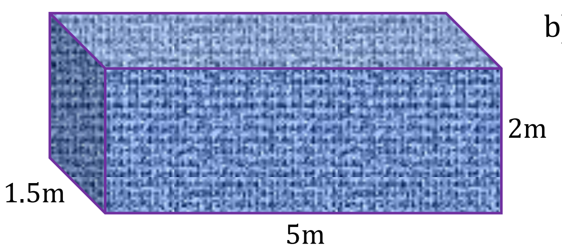
Find the volume of a cuboid whose measurements are 12cm by 11cm by 8cm.

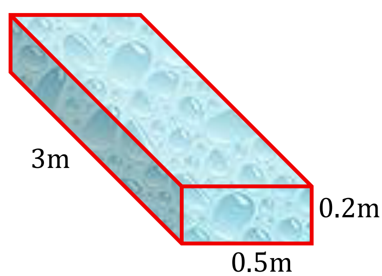
$$V = L \times W \times H$$
$$= (12 \times 11 \times 8)cm^3$$
$$= 1056cm^3$$

Exercise 11.

Work individually

1. Find the volume in cubic metres.

a) 

b) 

2. One cube measures 8cm, another cube measures 11cm. What is the sum of their volumes in cubic centimetres?

3. A rectangular container with a base area of $350m^2$ and a height of 20m is filled with juice. If the juice was packed into $250cm^3$ packets, how many packets were packed?
4. The volume of a rectangular tank is $72.9m^3$. The tank has a square base. If the height is 8.1 metres, what is the measure of one side of the square base?
5. A rectangular tank 45cm long and 25cm wide was $\frac{3}{4}$ full of water. What is the volume of water required to fill the tank?
6. A cube-shaped tank of 7.5m was full of water. After removing $14600cm^3$ of water the level of water become 5cm high. What was the height of the container?
7. A container with a volume of 0.09 cubic metres is full of water. The water is then poured into 15cubic centimeter containers. How many such containers are used?
8. One cube measures 0.8m. Another cuboid measures $1.2m \times 5m \times 0.3m$. What is the difference in their volume in cubic centimetres?
9. A company packs 250 packets of $750cm^3$ of pineapple juice while another packs 485 packets of $500cm^3$ of pineapple juice each day. How many cubic metres of juice the two companies pack in the month of October?

Volume of cylinders

Volume is the amount of space in a container.

Formula of calculating the volume of the cylinder is equal to

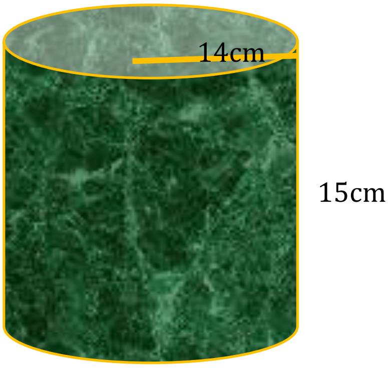
$$Volume = Base\ area \times height$$

$$= \pi r^2 \times h$$

$$= \frac{22}{7} r^2 \times h$$

Example 19.

Find the volume of the cylinder below.



$V = Base\ area \times Height$

$$\frac{22}{\cancel{7}} \times \overset{2}{\cancel{14}} cm \times 14cm \times 15cm$$

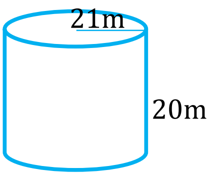
$$(22 \times 2 \times 14 \times 15)cm^3$$

$$= 9240cm^3$$

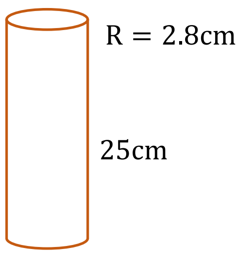
Activity 13:

1. Find the volume of the cylinders below. $\pi\ is\ approximately\ \frac{22}{7}$

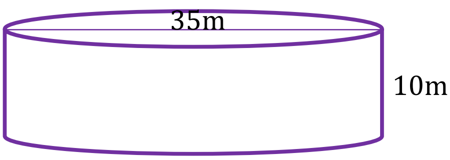
a)



b)



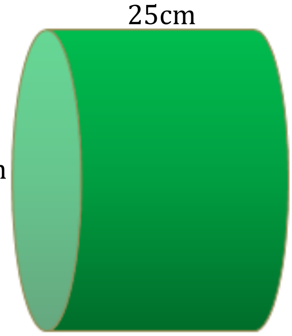
c)



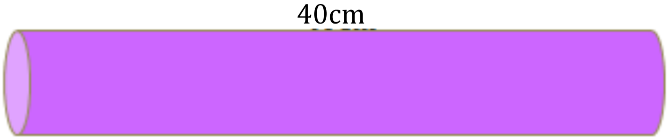
2. Find the volume of the cylinders below.

(i)

(ii)

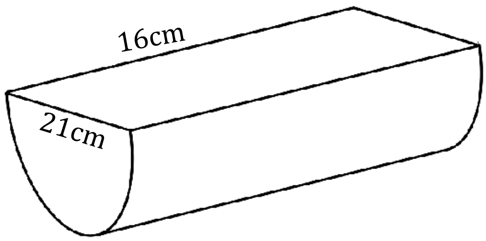


(ii)

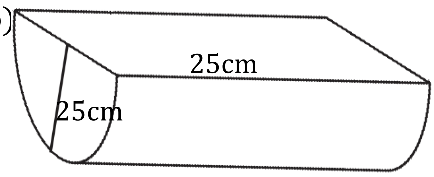


3. The figures below represent halves of cylindrical solids whose dimensions are shown. Find their volumes.

a)



b)



2.8 Problems involving Time, speed and distance

Formula

$Time\ taken = Distance \div Speed.$	$T = D \div S$
$Speed = Distance \div Time\ taken.$	$S = D \div T$
$Distance = Speed \times Time\ taken$	$D = S \times T$

Conversion of M/s to Km/h

Example 20.

Convert 15m/s to km/h

1000m = 1Km   15m =15÷1000km

3600sec = 1Hour   1Sec = 1÷3600 hour

Formula

$Speed = Distance \div (Time\ taken)$

$15 min\ in\ 1 sec = 15 \times 60 sec$

$15 \times 60 \times 60 m$ in 1 hour

$$\frac{15 \times 60 \times 60}{1000} km\ in\ 1\ hour$$

$$= 54 Km/h$$

NB 1: m/s ⟶ Km/h

Take speed in $m/s \times \frac{3600}{1000}$

NB 2: Km/h ⟶ m/s

Take speed in $km/h \times \frac{1000}{3600}$

Exercise 13.

Show how you got your answer

1. A cyclist took 18 minutes to travel from his home to school at a speed of 36Km/h. He took 20 minutes to travel back from school to his home. What was his average speed in M/s from school to his home?
2. A motorist left town C at 7.15a.m for town B a distance of 510km. He travelled the first 150km in $1\frac{1}{5}$ hours and stopped for15 minutes to take a cup of tea. He went on with the journey arriving in town D at 12.55p.m. What was his average speed for the whole journey?
3. A driver started on a journey of 450km at 7.30a.m travelling at an average speed of 90km/h. After travelling for 120km, he rested for 25minutes. He then continued with the rest of the journey at an average speed of 60km/h. At what time did he complete the journey?

4. In a relay race Faiza ran 100m which is $\frac{1}{3}$ of the race in 3 minutes. Mukami ran another 100m in 5 minutes while Cheromo ran the remaining part in 2 minutes. What was the average speed for the whole race in m/s?
5. Ayesha left home and walked for $1\frac{1}{3}$ hours at an average speed of 9km/h. She rested for 20 minutes and continued with the journey for 3hours at an average speed of $4\frac{2}{15}$km/h. What was the average speed for the whole journey?
6. Imran left town R at 7.15a.m for town S travelling at a speed of 75km/h, Saima left town S at 8.00a.m for town R at a speed of 9 km/h. The two met at a place 225km away from town R. What was the distance between town R and S?
7. A motorist driving at a speed of 80km/h was expected to arrive on time in town E 300km away. After driving for $2\frac{1}{4}$hours he rested for ¼ hour to take lunch. At what speed in km/h did he drive after taking lunch if he had to arrive at the expected time?
8. A Bus travelling from town M to town N at an average speed of 80km/h took 48minutes. Another bus took 40minutes to travel the same distance. What was the difference in their speed in km/h?
9. A motor bike travelled 450km at an average speed of 90km/h. On the return journey the average speed decreased to an average speed of 60km/h. Calculate the average speed in km/h for the whole journey.
10. Abdallah left home at 8.15a.m for Juba a distance of 300km. After driving for $2\frac{1}{4}$hours, he rested for 45minutes. He then continued with his journey and reached Kisumu at midday. What was his average speed for the journey?

Check your answers with your partner, explain how you worked it out.

2.9 Weight

The charts below will help you to convert between different metric units of weight.

METRIC WEIGHT CONVERSIONS					
1 gram	=	1000 milligrams	1g	=	1000 mg
1 decagram	=	10 grams	1dag	=	10g
1 kilogram	=	1000 grams	1 kg	=	1000 g
1 tonne (1 megagram)	=	1000 kilograms	1 tonne (1 Mg)	=	1000 kg
1 gigagram	=	1000 megagrams	1 Gg	=	1000 Mg

Exercise 14:

1. A tin of baked beans weighs 485g. How many grams less than 1.55kg will 2 tins of beans weigh?
2. The combined weight of 6 TV's is 138kg. How much does each TV weigh?
3. DVD players weigh 3kg and I buy 4 TV's and 4 DVD players. How much does my purchase weigh?
4. The limit of the baggage that each person can bring on an airplane is 20 kilograms. Achol's suitcase weighs 24 000 grams, and his brother Garang's weighs 23 500g. How much over the limit are their suitcases together?
5. To bake a 250g cake, you need to use 70 grams of butter.
   a) How much butter do you need to make a 2kg cake?
   b) If you use 280 grams of butter, how much does the cake weigh?
   c) If you use 560 grams of butter, what does the cake weigh?
6. Tim put a 0.975kg weight on one side of a set of balancing scales. William then put a 255g and a 300g weight on the other side.
   a) How much more does the William need to add to his side to make the scales balance?
   b) What does Tim need to add to his side of the scale to make it weigh 1560g?

2.10 Temperature

The temperature of an object is measured by an instrument called thermometer.

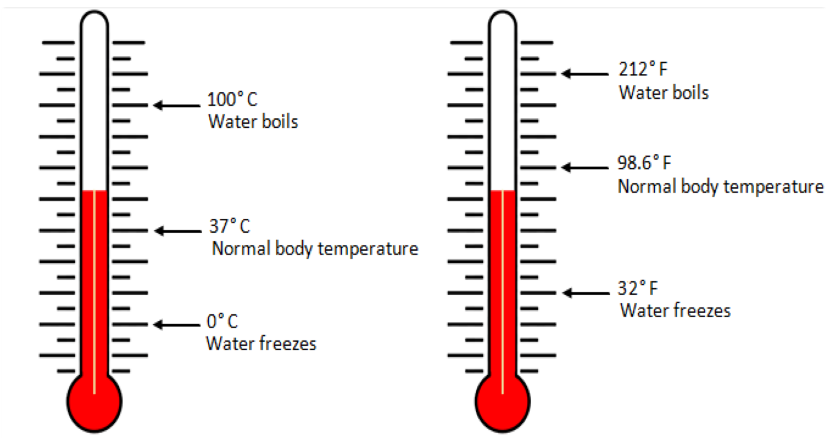
Activity 14

Take two cups, one containing cold water and another with warm water. Put your finger in one cup and of another hand in the other cup. Discuss the difference, what do you notice?

We found that, one cup contains cold water. But the question is how much cold and how much hot. To find this out, we need some measure of hotness or coldness.

Temperature is the degree of hotness or coldness of a body. The instrument which measures the temperature of body is known as **thermometer.**

Each thermometer has a scale. Two different temperature scales are in common use today:



Celsius Thermometer

Fahrenheit Thermometer

Thermometer has scale in degree Fahrenheit (°F) and in degree Celsius (°C). The Fahrenheit scale has the melting point of ice at 32° F and the boiling point of water at 212° F.
Thus, the Fahrenheit scale is marked from 32° to 212° where 32° F shows the freezing point of water and 212° F shows the boiling point of water. At present most of the countries use the degrees Celsius thermometers.
The Celsius scale (is also called centigrade scale) thermometer has 0° C as freezing point of water and 100° C as the boiling point of water.

Activity 15

In pairs, ask your partner the following questions.

1. The instrument used to measure body temperature is called.
2. The normal body temperature is.
3. The liquid inside the thermometer is called.
4. The units of measure of temperature are.
5. 0°C is cooler than 0°F.

Conversion of Temperature

In conversion of temperature from one scale into another the given temperature in °C we can convert it into °F and also the temperature in °F we can convert it into °C.

$$°F = \frac{°C \times 9}{5} + 32$$

$$C^0 = \frac{5}{9}(F^0 - 32)$$

The steps which are used in this conversion are given as:
1. When the temperature is given in degree Celsius:
Step I: Multiply the given temperature in degree by 9
Step II: The product we obtained from step I divide it by 5.
Step III: Add 32 with the quotient we obtained from step II to get the temperature in degree Fahrenheit.

$$\text{Temperature in degree Celsius} \xrightarrow[\text{by 9}]{\text{Multiply}} \xrightarrow[\text{by 5}]{\text{Divide}} \xrightarrow[\text{32}]{\text{Add}} \text{Temperature in °F}$$

2. When the temperature is given in degree Fahrenheit:
Step I: Subtract 32 from the given temperature in degree
Step II: The difference we obtained from step I multiply it by 5.
Step III: The product we obtained from step II divide it by 9 to get the temperature in degree Celsius.

Temperature in degree Fahrenheit $\xrightarrow[\text{32}]{\text{Subtract}}$ $\xrightarrow[\text{by 5}]{\text{Multiply}}$ $\xrightarrow[\text{by 9}]{\text{Divide}}$ Temperature in °C

Example 21.

1. Convert 50° C into degree Fahrenheit:
50° C

$$°F = \frac{°C \times 9}{5} + 32$$
$$°F = \frac{50 \times 9}{5} + 32$$
$$°F = \frac{450}{5} + 32$$
$$°F = 90 + 32$$
$$°F = 122$$

Therefore, 50° C = 122° F

2. Convert 212° Finto degree Celsius:
212° F

$$C^0 = \frac{5}{9}(F^0 - 32)$$
$$C^0 = \frac{5}{9}(212 - 32)$$
$$C^0 = \frac{5}{9}(180)$$
$$C^0 = \frac{5}{\cancel{9}}(\overset{20}{\cancel{180}})$$
$$C^0 = 100$$

Therefore, 212° F = 100° C

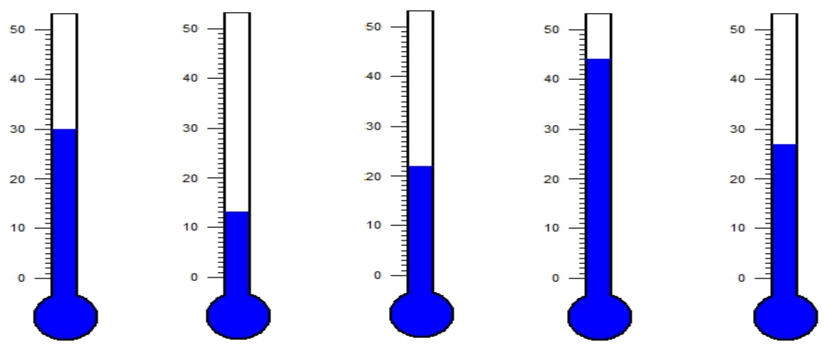
Play a game to understand Celsius and Fahrenheit

In groups of three, write and cut out the statements in the table below and share them out.
One learner calls out their card the person with the correct card holds it up.

	On the Celsius Scale	On the Fahrenheit Scale
Water freezes at	0°	32° F
Water boils at	100° C	212° F
Normal body temperature	37° C	98.6° F

Exercise 16:

1. Read and write the temperature shown on each thermometer in ^{0}C



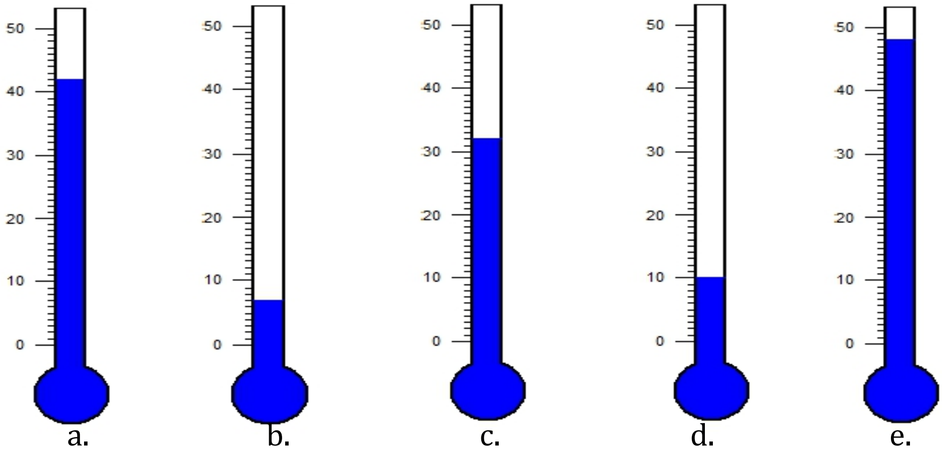
a. b. c. d. e.

2. Convert Celsius into Fahrenheit

a) 35^{0}C b) 20^{0}C c) 50^{0}C

d) 65^{0}C d) 90^{0}C e) 80^{0}C

3. Read and write the temperature shown on each thermometer in ^{0}F



4. Convert Fahrenheit into Celsius

a) 149^0F b) 95^0F c) 50^0F

d) 122^0F e) 41^0F f) 194^0F

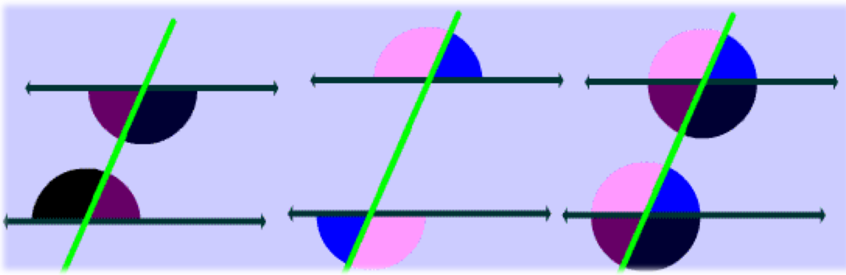
UNIT 3: GEOMETRY

3.1 Transversal lines and angles they form

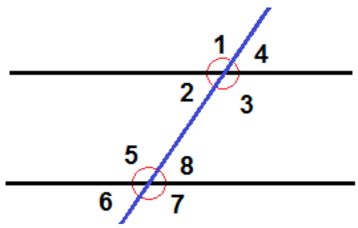
A **transversal** is a line that passes through two lines in the same plane at two distinct points.

There are 3 types of angles that are congruent: Alternate , and Corresponding Angles.

Alternate Alternate Corresponding



When a transversal intersects with two parallel lines eight angles are produced.



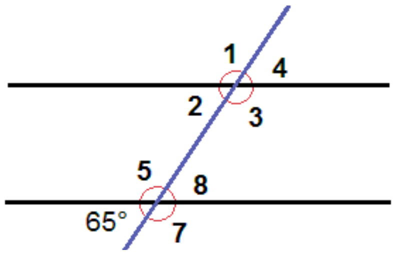
The eight angles will together form four pairs of **corresponding angles**. Angles 1 and 5 constitutes one of the pairs. Corresponding angles are congruent.

All angles that have the same position with regards to the parallel lines and the transversal are corresponding pairs e.g. 3 + 7, 4 + 8 and 2 + 6.

Angles that are in the area between the parallel lines like angle 2 and 8 above are called **interior angles** whereas the angles that are on the outside of the two parallel lines like 1 and 6 are called **exterior angles**.

Angles that are on the opposite sides of the transversal are called alternate angles e.g. 1 + 8.
All angles that are either exterior angles, interior angles, alternate angles or corresponding angles are all congruent.

Example 1.



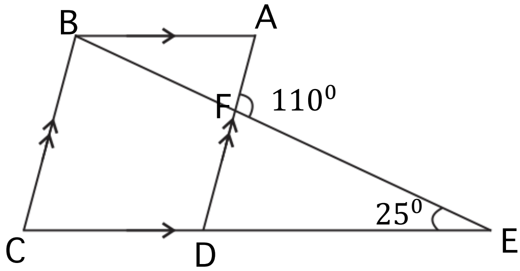
The picture above shows two parallel lines with a transversal. The angle 6 is 65°. Is there any other angle that also measures 65°?

Solution

6 and 8 are vertical angles and are thus congruent which means angle 8 is also 65°.
6 and 2 are corresponding angles and are thus congruent which means angle 2 is 65°.
6 and 4 are alternate exterior angles and thus congruent which means angle 4 is 65°.

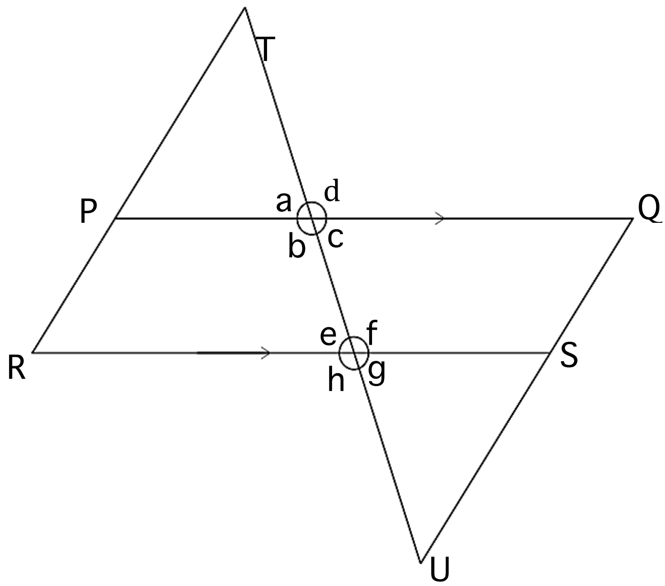
Exercise 1:

1. Figure ABCD shown below is a parallelogram. Line CDE is a straight line, angle DEF= 25˚ and angle EPA= 100˚.



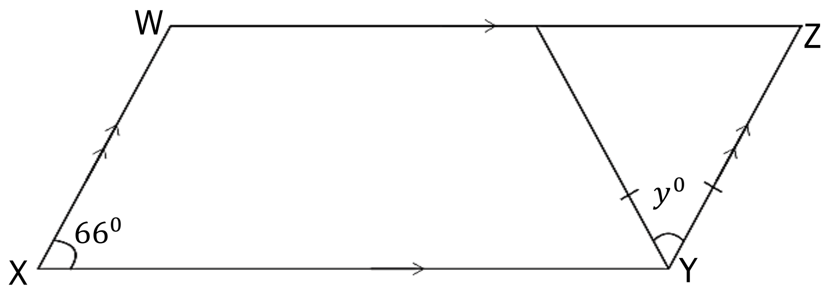
What is the size of angle EBC?

2. In the figure below, line PQ is parallel to line RS. Lines TR and TU are transversals.



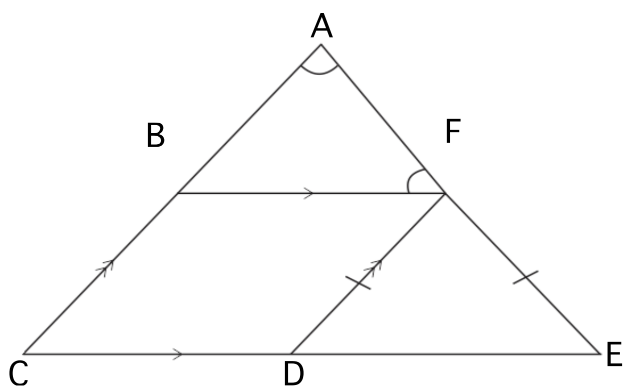
Write the correct statements about angles a, b, c, d, e, f, g and h?

3. What is the measure of the exterior angle XYZ in the quadrilateral WXYZ below drawn to scale?



What is the value of angle y?

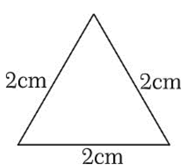
4. The figure below is made up of a parallelogram BCDF, triangles DFE and ABF, angle BAF= 80°and angle BFA= 52°



What is the value of angle DFE?

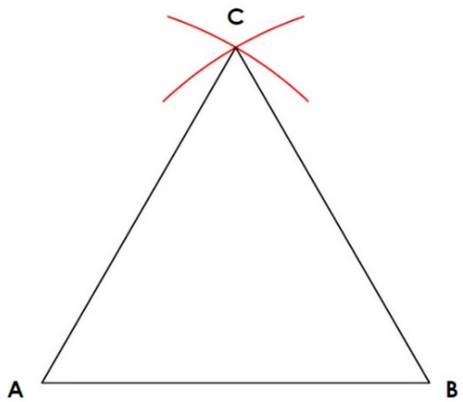
3.2 Types of triangles based on sides

Equilateral triangle: A triangle having all the three sides of equal length is an equilateral triangle.



Since all sides are equal, all angles are equal too.

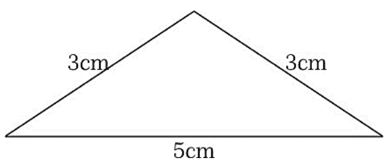
How to construct an equilateral triangle



By setting your compass to radius AB and swinging two arcs from Point A and point B, you will create point C.
Join point A to C and B to C. This will create an Equilateral triangle.

Activity 1
In groups, draw an equilateral triangle using the above steps.

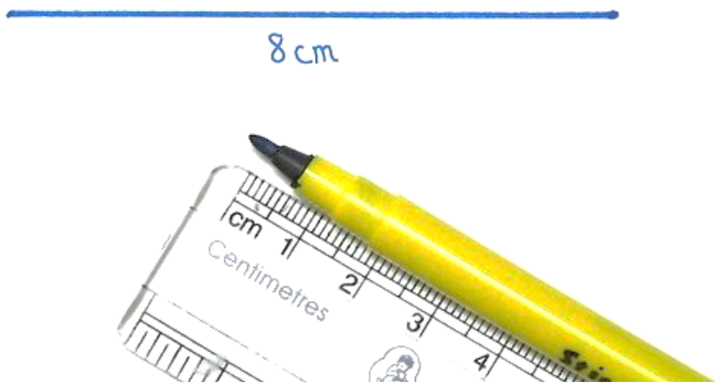
Isosceles triangle: A triangle having two sides of equal length is an Isosceles triangle.



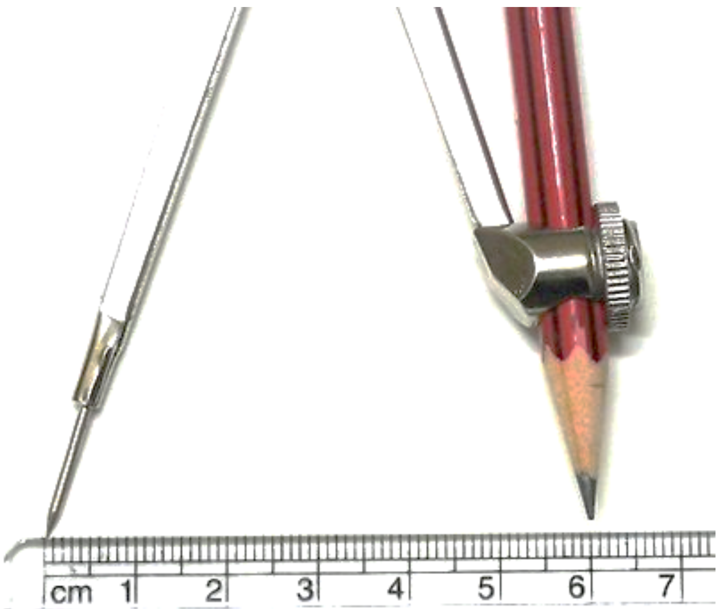
The two angles opposite to the equal sides are equal.

How to Construct an Isosceles Triangle

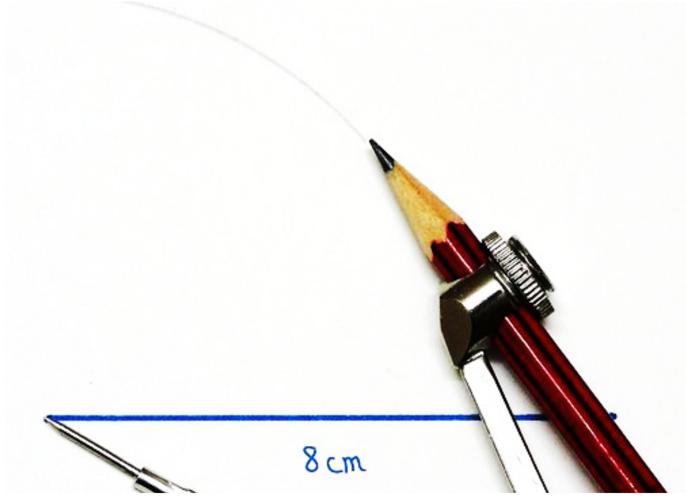
Using a protractor, you can use information about angles to draw an isosceles triangle.
We should know the length of the triangle's base and the length of the two equal sides.



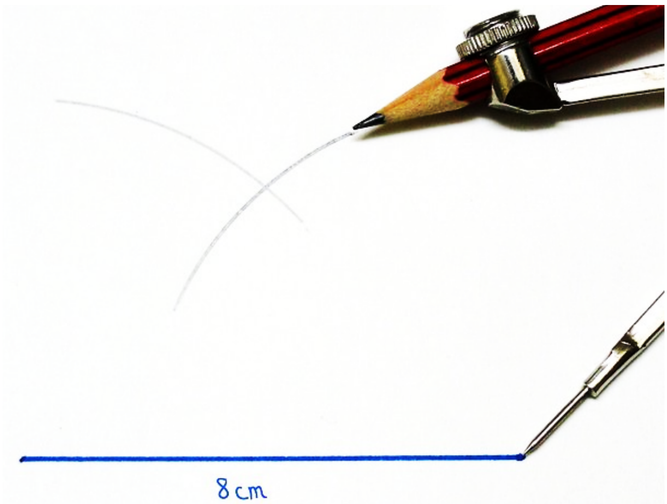
Draw the base. Use a ruler to make sure that your line is measured exactly. For example, if you know that the base is 8 cm long, use a pencil and a ruler to draw a line exactly 8 cm long.



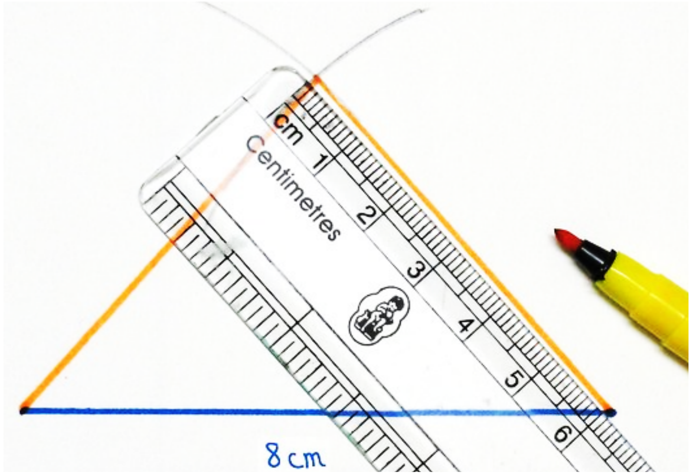
Set the compass. To do this, open the compass to the width of the equal side lengths. If you are given the measurement, use a ruler.
For example, if the side lengths are 6 cm, open the compass to this length.



Draw an arc above the base. To do this, place the tip of the compass on one of the base's endpoints. Sweep the compass in the space above the base, drawing an arc. Make sure the arc passes at least halfway across the base.



Draw an intersecting arc above the base. Without changing the width of the compass, place the tip on the other endpoint of the base. Draw an arc that intersects the first one.

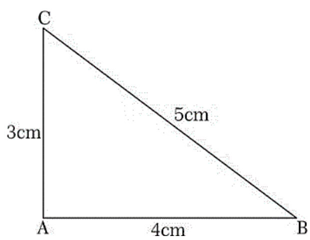


Draw the sides of the triangle. Use a ruler to draw lines connecting the point where the arcs intersect to either endpoint of the base. The resulting figure is an isosceles triangle.

Activity 2

In groups draw an isosceles triangle using the above steps.

Right-angled triangle: A triangle whose one angle is a right-angle is a Right-angled triangle.



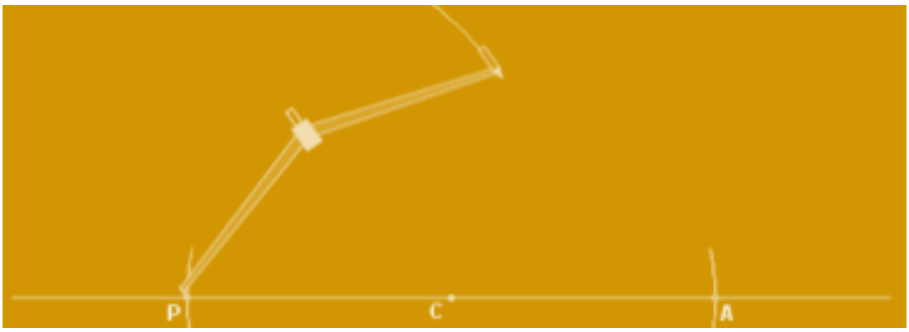
Constructing a Right Angled Triangle

The requirements for the construction are a ruler and a compass. Let us construct a right-angled triangle ABC, right angled at C. Consider the length of the hypotenuse AB = 5 cm and side CA = 3 cm. The steps for construction are:

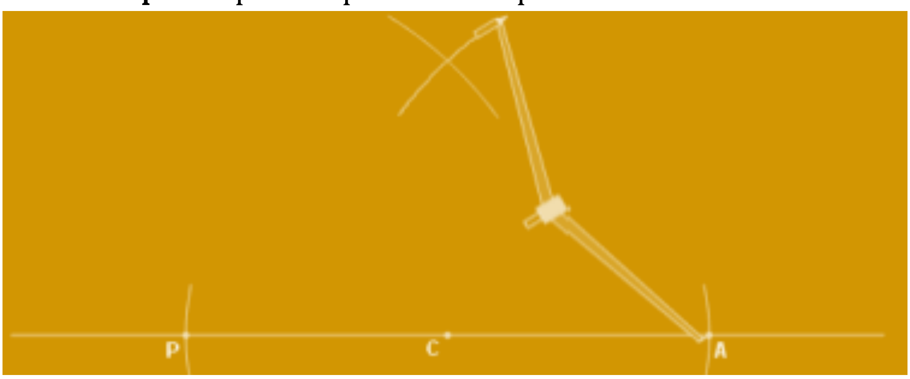
- **Step 1:** Draw a horizontal line of any length and mark a point C on it.

- **Step 2:** Set the compass width to 3 cm.
- **Step 3:** Place the pointer head of the compass on the point C and mark an arc on both the sides of C.

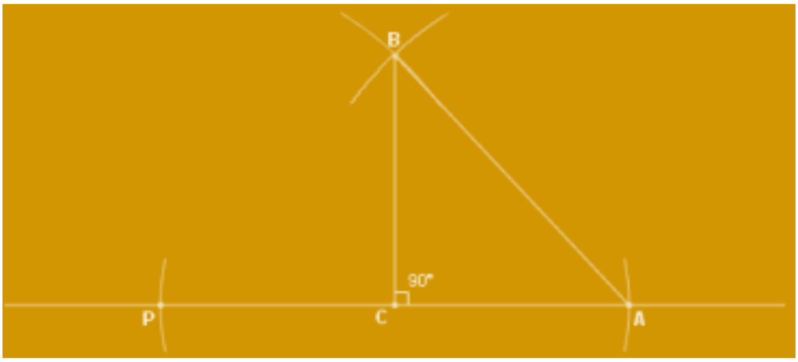
- **Step 4:** Mark the points as P and A where the arcs cross the line.
- **Step 5:** Set the compass width to the length of the hypotenuse, that is, 5 cm.
- **Step 6:** Place the pointer head of the compass on the point P and mark an arc above C.



- **Step 7:** Repeat step 6 from the point A.



- **Step 8:** Mark the point as B where the two arcs cross each other.
- **Step 9:** Join the points B and A as well as B and C with the ruler.



We obtain a right-angled triangle ACB of the required measurements.

Activity 3

In pairs, draw a right angled triangle using the above steps.

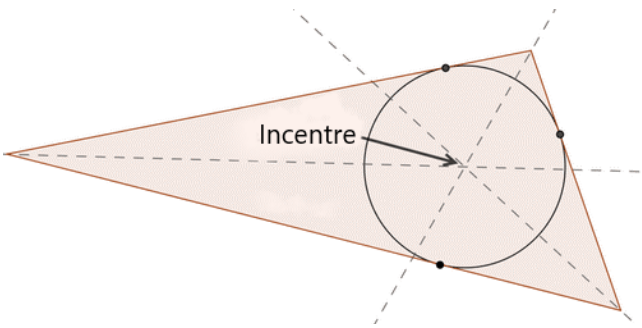
3.3 Inscribing and circumscribing circles of triangles

Given a triangle, what's the difference between the **inscribed circle** of the triangle and the **circumscribed circle** of the triangle?

Inscribing a circle

Given a triangle, an **inscribed circle** is the largest circle contained within the triangle. The inscribed circle will touch each of the three sides of the triangle in exactly one point. The center of the circle inscribed in a triangle is the **incentre** of the triangle, the point where the angle bisectors of the triangle meet.

NB: The inscribed circle of a triangle is inside the triangle.

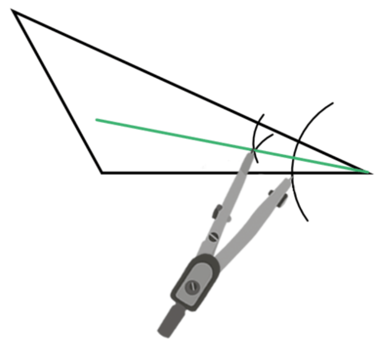


Steps to construct the inscribed circle:

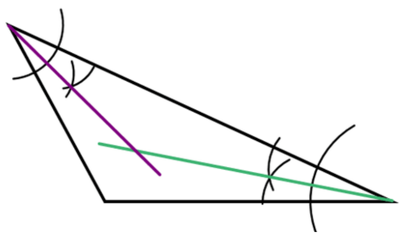
Bisect the angles of the triangle and produce them such that they intersect somewhere within the circle. The point of intersection is known as the incentre.

Draw a triangle. Construct the angle bisectors of two of its angles. Why is the point of intersection of the two angle bisectors the incentre of the circle?

Use your Pair of compasses and straightedge to construct the angle bisector of one of the angles.



Repeat with a second angle.



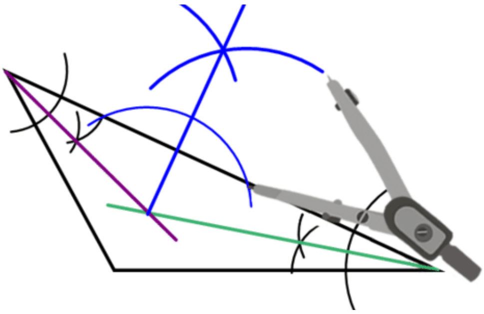
The point of intersection of the angle bisectors is the incentre.

It is not necessary to construct all three angle bisectors because they all meet in the same point. The third angle bisector does not provide any new information.

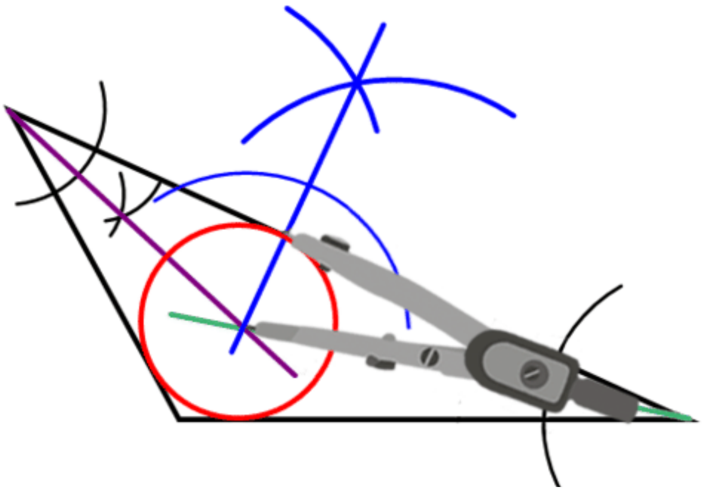
The segment connecting the incentre with the point of intersection of the triangle and the bisector is the radius of the circle.

Construct a circle centered at the incentre that passes through the point of intersection of the side of the triangle and the perpendicular line from the problem above.

Construct a line perpendicular to one side of the triangle that passes through the incentre of the triangle.



Use your Pair of compasses and straightedge to construct a line perpendicular to one side of the triangle that passes through the incentre.



Note that this circle touches each side of the triangle exactly once.

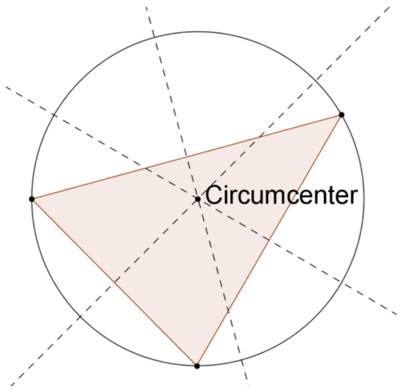
Activity 4:

In groups, do the activity

1. Draw a triangle and construct the angle bisector of two of its angles.
2. Continue with your triangle from 1. Construct a line perpendicular to one side of the triangle that passes through the incentre of the triangle.
3. Continue with your triangle from 1 and 2. Construct the inscribed circle of the triangle.
4. hy is it not necessary to construct the angle bisector of *all three* of the angles of the triangle?
5. Explain why the incentre is equidistant from each of the sides of the triangle.

Circumscribing a circle

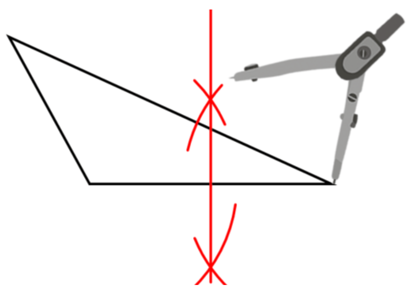
Given a triangle, the circumscribed circle is the circle that passes through all three vertices of the triangle. The center of the circumscribed circle is the circumcenter of the triangle, the point where the perpendicular bisectors of the sides meet.

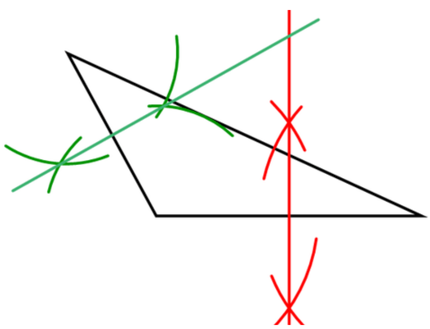


Steps to construct the circumscribed circle:

Bisect the sides of the triangle and produce them such that they intersect somewhere within or outside the circle. The point of intersection is known as the circumcenter.

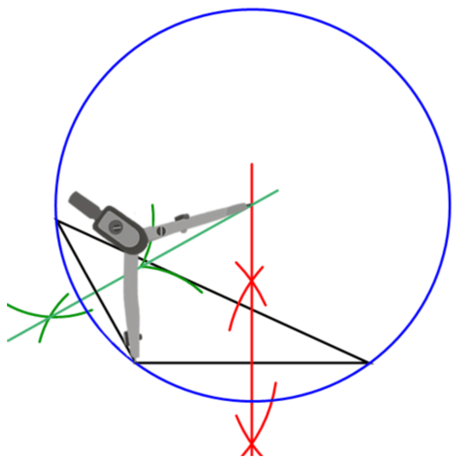
Use your Pair of compasses and straightedge to construct the perpendicular bisector of one side.





Repeat with a second side.

The point of intersection of the perpendicular bisectors is the circumcenter. It is not necessary to construct all three perpendicular bisectors because they all meet in the same point. The third perpendicular bisector does not provide any new information.
Construct a circle centered at the circumcenter that passes through one of the vertices of the triangle. This circle should pass through *all three* vertices.



Activity 5:

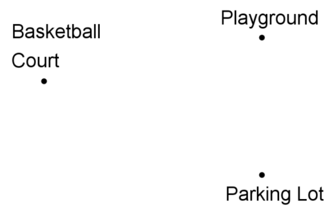
In groups, do the activity

1. Draw a triangle and construct the perpendicular bisector of two of its sides.
2. Continue with your triangle from 1. Construct the circumscribed circle of the triangle.
3. Explain why the circumcenter is equidistant from each of the vertices of the triangle.

Exercise 2.

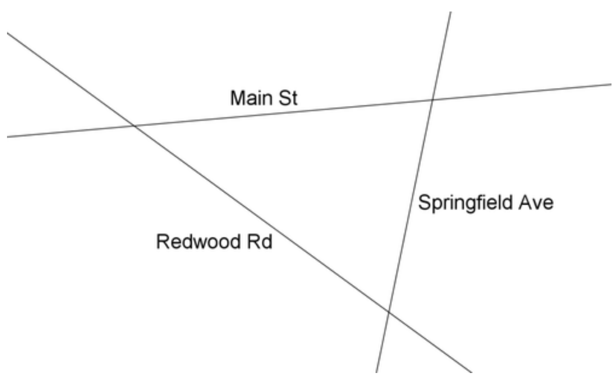
Work in groups;

1. You are a vendor selling food from a food truck at a local park. You want to position your truck so that it is the same distance away from each of the three locations shown on the map below.



   a. Is the point of interest the incentre or the circumcenter? How do you know, explain your thinking to the group.
   b. Find the point on the map that is equidistant from each of the three locations.

2. A new elementary school is to be constructed in your town. The plan is to build the school so that it is the same distance away from each of the three major roads shown in the map below.



   a. Is the point of interest the incentre or the circumcenter?
   b. Find the point on the map that is equidistant from each of the three roads.

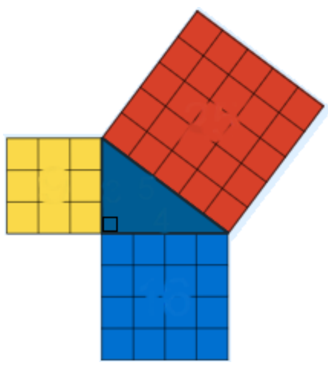
Activity 6:

1. Draw two triangles of different shapes and then construct the circle that circumscribes them. Next, draw two triangles and then construct the circle that inscribes them.
2. Construct a triangle PQR such that lines QR= 4.5cm, QP= 6.9cm and angle PQR= 100˚. Construct a circle touching the three vertices. What is the radius of the circle?
3. Construct triangle ABC in which line AB= 6cm, CA= 9cm and angle BCA= 140˚. Draw a circle that touches the three sides of the triangle. What is the length of the radius of the circle?

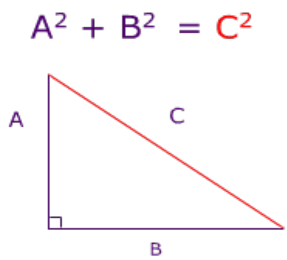
3.4 Pythagoras' Theorem

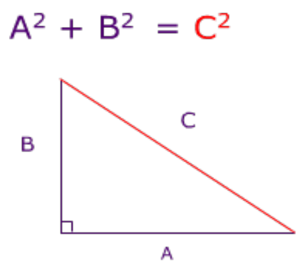
What is the Pythagoras' Theorem?

The Pythagorean Theorem or Pythagoras' Theorem is a formula relating the lengths of the three sides of a **right angled triangle.**



A right angled triangle consists of two shorter sides and one side called the **hypotenuse.** The hypotenuse is **the longest side** and is **opposite the right angle.**



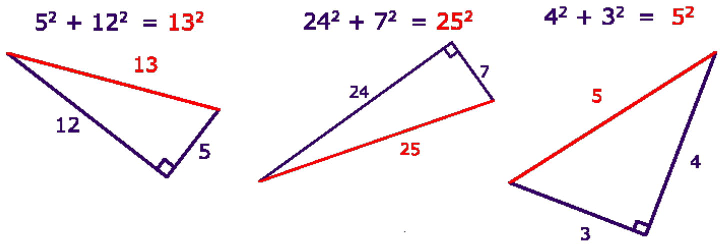


If we take the length of the **hypotenuse** to be c and the length of the **other sides** to be a and b then this theorem tells us that:

$$c^2 = a^2 + b^2$$

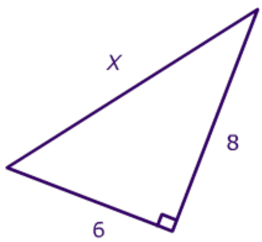
Pythagoras' Theorem states that; ***'In any right angled triangle, the sum of the squared lengths of the two shorter sides is equal to the squared length of the hypotenuse.'***

Pythagoras' theorem only works for **right angled triangles.**
When you use the Pythagorean Theorem, just remember that the hypotenuse is always 'C' in the formula above.
Look at the following triangles to see pictures of the formula.



Example 2.

Use the Pythagorean Theorem to determine the length of X



Step 1
Identify the shorter and the hypotenuse of the right triangle.
The shorter have length 6cm and 8cm. X is the hypotenuse because it is opposite the right angle.
Step 2
Substitute numbers into the formula (remember 'C' is the hypotenuse)
$A^2 + B^2 = C^2$
$6^2 + 8^2 = X^2$
Step 3

Solve for the unknown

$$A^2 + B^2 = C^2$$

$6^2 + 8^2 = X^2$

$36 + 64 = X^2$

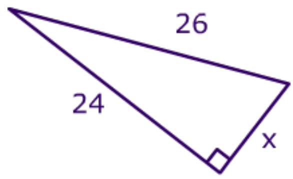
$100 = X^2$

$X = \sqrt{100}$

$X = 10cm$

Example 3.

Use the Pythagorean Theorem to determine the length of X



Step 1

Identify the length and width and the hypotenuse of the right triangle. The length have length 24cm and x is width. The hypotenuse is 26cm.

Step 2

Substitute values into the formula.

$A^2 + B^2 = C^2$

$X^2 + 24^2 = 26^2$

$X^2 + 576 = 676$

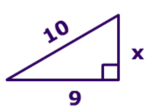
$X^2 = 100$

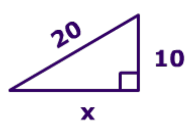
$X = \sqrt{100}$

$X = 10cm$

Exercise 3.

1. Calculate the value of X.

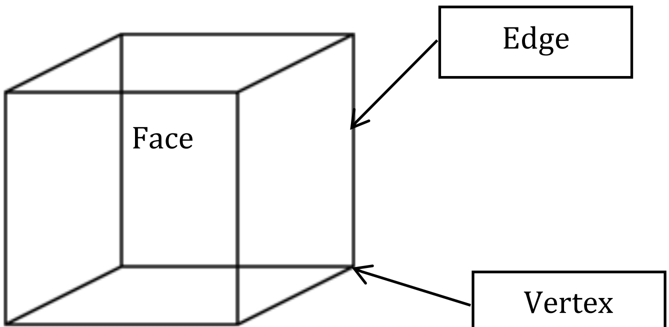
a. 

b. 

2. A wheelchair ramp is needed at the entrance to a building. There is only 10 metres of space available for the ramp. How long should the ramp be?
3. A roof is being placed on a frame that is 9 metres tall and 30 metres wide. How long are the diagonal pieces of the frame?
4. Which one of the following sets of measurements can be used to construct a right-angled triangle?
   (A) 4.5cm, 6cm, 9cm (B) 3.75cm, 5.25cm, 6cm
   (C)2.25cm, 3cm, 3.75cm (D) 5.25cm, 9cm, 11.25cm
5. A painter used a ladder to paint a wall of a storey building which was 24metres high. The ladder was placed 7metres away from the wall. What was the height of the ladder?

3.5 Pyramids and prisms

Recall from level 3

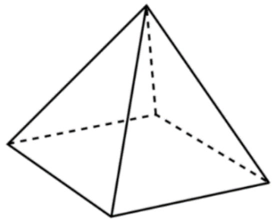


Faces are flat shapes
Edges are lines where faces meet
Pyramids and prisms are two different shapes. The main difference between a pyramid and prism is the fact that a prism has two bases, while the pyramid only has one.
The type of pyramid is determined by the base. For example: a triangular pyramid will have a triangular base, while a square pyramid will have a square base, and so on.
The type of prism is determined by the shape of the base. For example: a triangular prism will have triangular bases, while a rectangular prism will have rectangular bases, an octagonal prism will have octagon bases, and so on.

Activity 7:

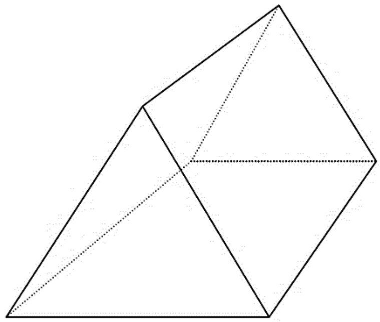
Collect different objects with the shapes below and use them to investigate.

1. The diagram below represents a triangular square based pyramid
   What is the total number of?
   Faces
   Vertices
   Edges



2. The diagram below represents a triangular prism
   What is the total number of?

   Faces
   Edges
   Vertices

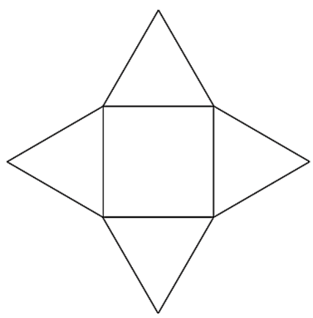


Exercise 4.

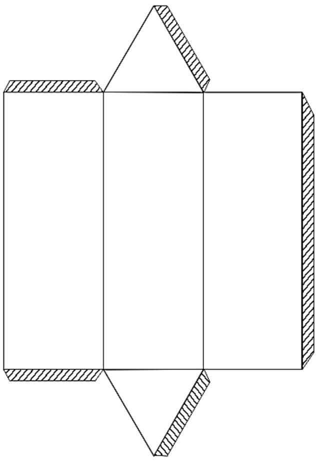
Work in pairs, draw the patterns below, cut them out and fold them along the lines.

1. The figure below shows a net made up of a square and 4 equilateral triangles

   If folded which solid can be formed from this net?

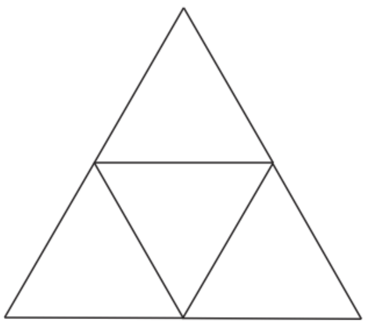


2. Below is a net of a solid. The shaded parts are to be folded and glued.



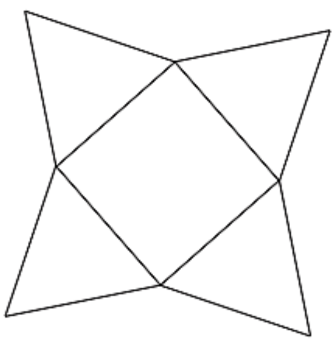
Which solid can be formed from the net?

3. The figure below represents the net of a solid.



The net is folded to form a solid. Which solid can be formed from the net?

4. The figure below represents the net of a solid.

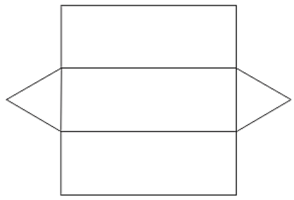


The net is folded to form a solid. How many edges will the solid have?

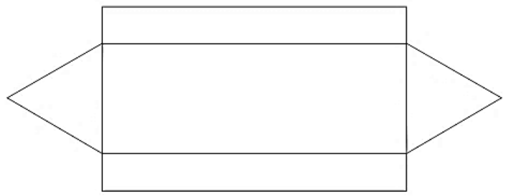
Is it a pyramid or prism?

5. Which one of the following figures is the net of a triangular prism?

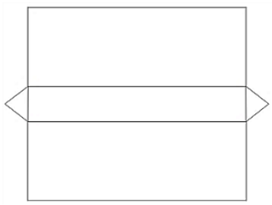
A.



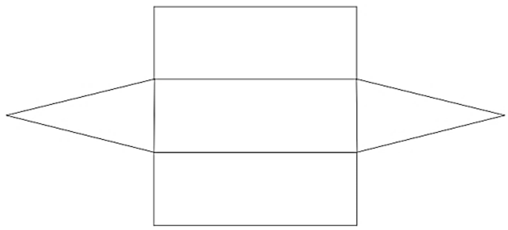
B.



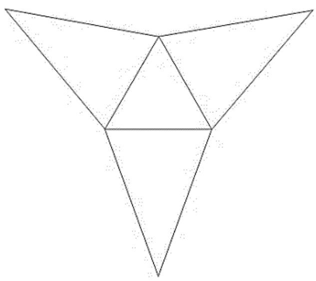
C.



D.



6. The figure represents the net of a solid



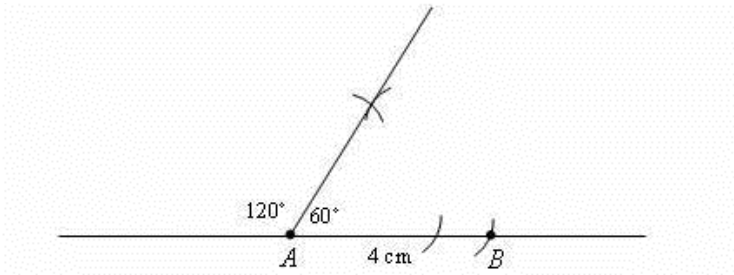
The net is folded to form the solid. How many vertices will the solid have?

3.6 Constructing a Parallelogram

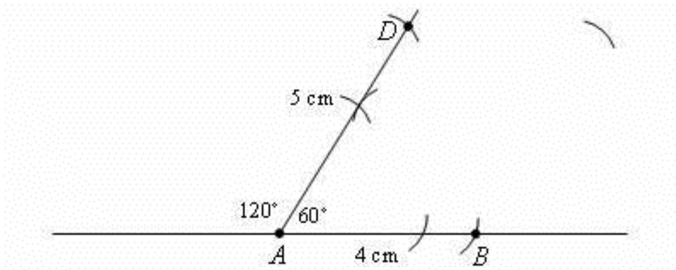
Construct a parallelogram $ABCD$ with sides $AB = 4$ cm and $AD = 5$ cm and angle $A = 60^\circ$. In pairs, follow the steps below.

Steps

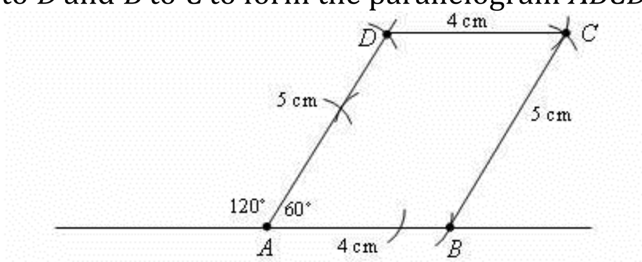
Step 1: Construct a line segment $AB = 4$ cm. Construct a 60° angle at point A.



Step 2: Construct a line segment $AD = 5$ cm on the other arm of the angle. Then, place the sharp point of the compasses at *B*and make an arc 5 cm above *B*.

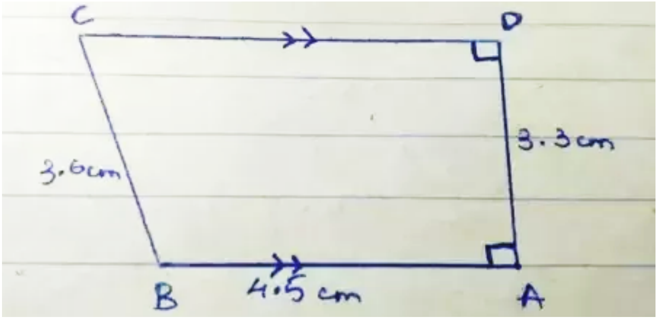


Step 3: Stretch your compasses to 4 cm, place the sharp end at *D* and draw an arc to intersect the arc drawn in step 2. Label the intersecting point *C.* Join *C* to *D* and *B* to *C* to form the parallelogram *ABCD*.



3.7 Constructing a trapezium

This is the figure of the trapezium.

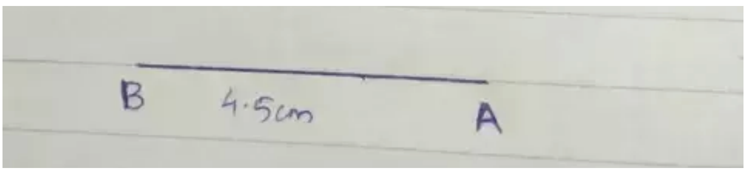


AB and CD are parallel.
The height of the trapezium is AD = 3.3cm.
BC= 3.6 cm

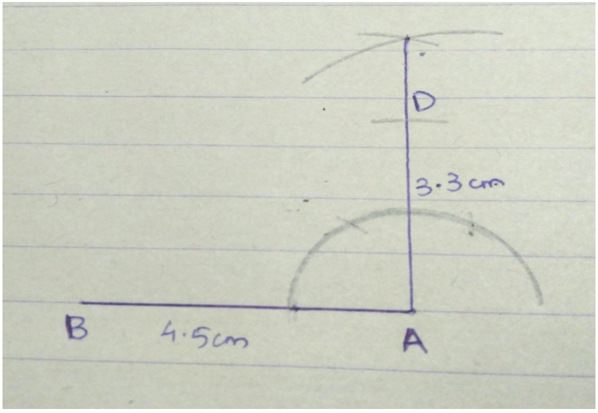
Example 4.

In pairs follow the steps to draw a trapezium.

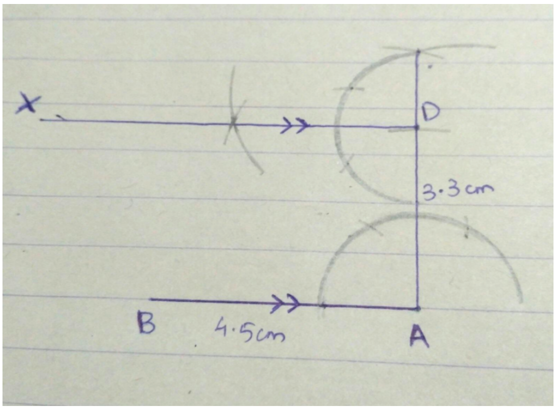
Draw a line AB=4.5 cm.



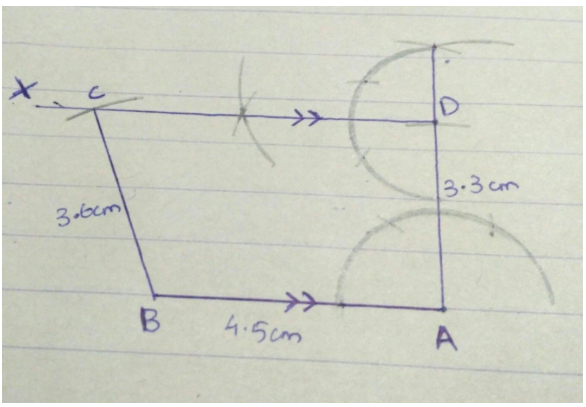
Next using the compass construct an angle BAD=90°. Since, the distance between the parallel sides is 3.3cm, it implies that the line perpendicular to AB at A is 3.3 cm. So, cut an arc of 3.3 cm on the perpendicular line and name it D.



The two sides (AB and CD) are parallel. Therefore, construct an angle ADX=90°. Draw a line DX which is parallel to AB.



Now we have three sides of the trapezium. So, cut an arc of 3.6 cm from B on line DX. Join B and C.



Thus, a trapezium with sides AB=4.5 cm, AD=3.3 cm, BC=3.6 cm and Angle B is obtuse.

3.8 Scale

Scale is the ratio of the length in a drawing (or model) to the length of the real things.

Real Horse
1500 mm high
2000 mm long

Drawn Horse
150 mm high
200 mm long

In the drawing anything with the size of "1" would have a size of "10" in the real world, so a measurement of 150mm on the drawing would be 1500mm on real life.

Example 5.

A road 3km is represented by a line 8cm long. What is the length of a road 12cm long?

$$8cm\ = 3km$$
$$8cm\ = 3000m$$
$$\therefore\ 1cm = ?$$

Cross multiply $\dfrac{1\times 3000}{8}$

$\therefore$ $1cm$ is $375m$

1cm rep 0.375km

$\therefore$ $12cm$ Will be 0.375 $\times$12

$\underline{\underline{= \mathbf{4.4km}}}$

Activity 8:

In groups measure the following and use the scale 1:100 to draw them on a paper.

a. School Compound.
b. Class room.
c. Playing ground.

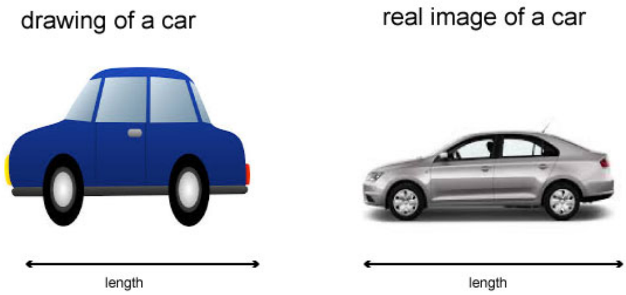
Exercise 5.

In groups, work out problems involving scale drawings.

1. A section of a tarmac road measures 8.5km, if it is drawn on a map it measures 17cm. What was the scale used?
2. A river measuring 5cm on a map has a length of 75km. What is the scale used on the map?
3. A map whose scale was 1:100000, an actual length of a right-angled plot of land measures 70m by 50m. What was the area of the triangular plot on the map?
4. On a map, 7.5cm represents 90km of the border of a certain county. What is the scale used?
5. A map is drawn to a scale of 1:15000. What is the distance in kilometres of a road which is 13.5cm on a map?
6. In scale drawing 1cm on a map represents an actual length of 25m. What area in the drawing will represent an actual area of 1 hectare?
7. A road is represented on a map by 3cm. What is the actual length of the road in kilometres if the scale used is 1:125000?
8. On a map of a scale of 1:100000, a rectangular shaped estate in town measures 9cm by 4cm. What is the actual length of the plot in kilometres?
9. A map is drawn to a scale of 1:800. What is the distance in metres of a road which is 17cm on the map?
10. A distance of 72km is represented on a scale drawing by a line measuring 18cm. What is the scale used in ratio form?

3.9 Drawing and interpreting linear scale

Given a scale diagram we can make drawing measurements.
Similarly, real (actual) measurements can be made on a real object.
Linear scale shows, the relationship between drawing (scale) length and actual length.
Linear scale of a diagram is given in statement or ratio form.
When the ratio form is used the unit of measurement for the drawing length and actual length is the same.



This is scale drawing.

It is not always possible to draw on paper the actual size of real-life objects such as the real size of a car, an airplane, house we need scale drawings to represent the size like the one you have seen.

Example 6.

In a scale diagram 1cm represents 50 m
(a) Write the scale in ratio form.
(b) Find the drawing length for 1.25 km.
(c) Find the actual length in kilometers corresponding to a length of 10.5cm on the diagram.

Solution

(a) 1cm represents 50cm = 50 x 100 c = 5 000cm

Ratio from is 1:5000

(b) 1.25 km = 1.25 x 1000m = 1 250m

Drawing length for 50m is 1cm
drawing length for 1 250m is $\frac{1\,250}{50}$ = 25 m

(c) 10.5 cm represents 10.5 x 50 cm = 525m = 0.525km

Activity 9

In a diagram of a plan of a factory a length of 2.5 cm represents an actual length of 12.5 m

(a) Work out the linear scale in ratio form.

(b) Find the distance on the plan between two buildings which are 35 m apart.

Conversions to recall

1 km =1000 m
1 km = 100,000 cm
1 m = 100 cm
1 ha = 10,000 m^2

Why are scale drawings important?

Careers that involve construction, architecture, city planning, design and map-reading all require knowledge of scale drawings

You also need to be able to use scale-drawings when you are travelling. (Maps)

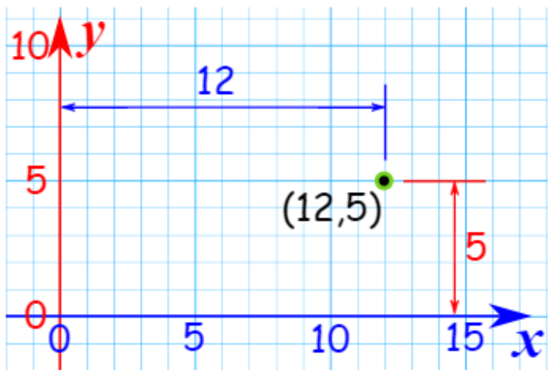
3.10 Coordinates

Coordinates are a set of values that show an exact position.

On graphs it is common to have a pair of numbers to show where a point is: the first number shows the distance along and the second number shows the distance up or down.

Example 7.

The point (12,5) is 12 units along, and 5 units up.



On maps the two coordinates often mean how far North/South and East/West.
There are other types of coordinates, too, such as polar coordinates and 3 dimensional coordinates.

Plotting points on a Cartesian plane

Just like with the Number Line, we can also have negative values.
Negative: start at zero and head in the opposite direction:
Negative x goes to the left, negative y goes down
For example (-6,4) means:
go back along the x axis 6 then go up 4.

And (-6,-4) means:
go back along the x axis 6 then go down 4.

Four Quadrants

When we include negative values, the x and y axes divide the space up into 4 pieces: Quadrants I, II, III and IV

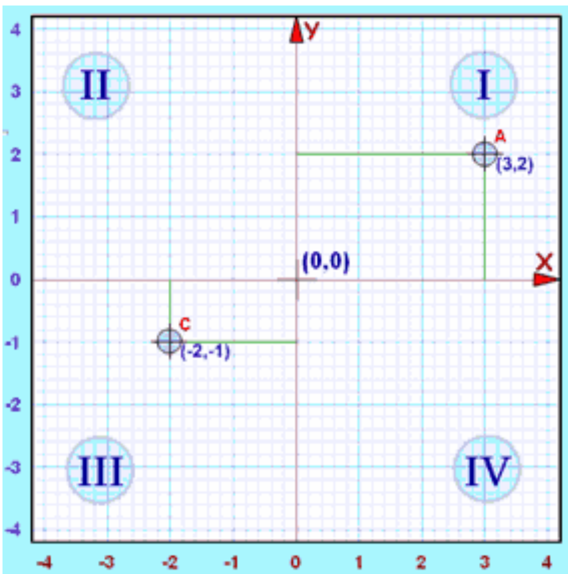
In Quadrant I both x and y are positive, but in Quadrant II x is negative (y is still positive), in Quadrant III both x and y are negative, and in Quadrant IV x is positive again, while y is negative.

Like this:

Quadrant	X (horizontal)	Y (vertical)	Example
I	Positive	Positive	(3,2)
II	Negative	Positive	
III	Negative	Negative	(-2,-1)
IV	Positive	Negative	

(They are numbered in a counterclockwise direction)

Example 8.



The point "A" (3,2) is 3 units along, and 2 units up.

Both x and y are positive, so that point is in "Quadrant I"

Example: The point "C" (-2,-1) is 2 units along in the negative direction, and 1 unit down (i.e. negative direction).

Both x and y are negative, so that point is in "Quadrant III"

Note: The word Quadrant comes from ***quad*** meaning four. For example, four babies born at one birth are called ***quadruplets*** and a ***quadrilateral*** is a four-sided ***polygon***.

UNIT 4: ALGEBRA

4.1 Simplifying algebraic expressions

In algebra, letters are used to represent unknown numbers. To simplify an algebraic expression, we collect like terms together.

Example 1.

Simplify $21y + 5 - 13y + 8$

$21y - 13y + 5 + 8$ Collect like terms together.

$8y + 13$ Simplify.

Activity 1

In groups, simplify the following algebraic expressions. Talk in your groups how you have done this.

a) $15x + 6 - 4x + 3$

b) $4 + 13p - 2 + 3p$

c) $12q - 5 + 5q + 8 - 3y$

d) $5 + 20s + 4 - 12s$

e) $18r + 6 - 12xy + 5r + 9$

f) $21p + 9 + 8p - 4 + 12xy$

Example 2.

Simplify $2(xy + 2) - 4(y + 3)$

$2xy + 4 - 4y + 12$ First open the brackets by multiplying by the number outside the bracket.

$2xy + 4 + 12 - 4y$ Collect the like terms together.

$2xy + 16 - 4y$ Simplify.

Activity 2

In groups, simplify the following equations showing your working out.

a) $3\ (p\ +\ 2)\ +\ 2\ (4p\ +\ 1)$

b) $q\ (2\ +\ 4)\ -4\ (5\ +\ 2p)$

c) $4\ (r\ +\ 2)\ +2\ (r\ -\ 2)$

d) $5\ (s\ +\ 6)\ +3\ (2s\ -\ 5)$

e) $4\ (5q\ +\ 2)\ -2\ (3\ +\ 2q)$

f) $3\ (6z\ -\ 5)\ +4\ (2z\ -\ 3)$

Example 3.

i. $y + y + y + y + y$ is put together as $5 \times y = 5y$.

ii. $m + m + m + m + m + m = 6 \times m = 6m$

When simplifying algebraic expressions, similar letters should be put together.

1. The operation sign (+ or –) placed before a term is for that term and determines the operation to be performed.
    i. + 7n means add 7n; - 9a means subtract 9a.
    ii. 12a – 7a means subtract 7a from 12a which results to 5a.
    $= 13y - 4y = 9y$
    iii. $p + 2p + 3p - 4p = 2p$
    iv. $3n + 7m + 15n - 4m - 11n$

    Putting like terms together we get;

    $$= (3n + 15n - 11n) + (7m - 4m) = 7n + 3m$$

    $$= 2\frac{1}{2}p + 4\frac{1}{4}q - \frac{3}{4}p - 2\frac{1}{2}q$$

    Putting like terms together

    $$= (2\frac{1}{2}p - \frac{3}{4}p) + (4\frac{1}{4}q - 2\frac{1}{2}q)$$

    $$= 1\frac{3}{4}p + 2q$$

    v. $\frac{1}{2}(5x + 7y) + \frac{1}{4}(3x - 6y)$

    Multiply every term in the brackets by the value outside the bracket.

$$= \frac{5}{2}x + \frac{7}{2}y + 3/4x - 6/4y$$

Putting like terms together

$$= \frac{5}{2}x + \frac{3}{4}x + \frac{7}{2}y - \frac{3}{2}y = \frac{13}{4}x + \frac{4}{2}y$$

$$= \frac{13}{4}x + 2y$$

vi. $\frac{15a + 12b - 6a + 8b}{4(3a - 2b) + 13b}$

$$= \frac{15a - 6a + 12b + 8b}{12a - 8b + 13b}$$

$$= \frac{9a + 20b}{12a + 13b - 8b}$$

$$= \frac{9a + 20b}{12a + 5b}$$

Activity 3:

Simplify the algebraic expressions below:

i. $d + 3d - 5d + 7d$

ii. $\frac{1}{2}(18x + 24y) - \frac{1}{4}(20x + 16y)$

iii. $\dfrac{\frac{3}{5}(15a + 20b) - \frac{2}{3}(9a - 6b)}{\frac{3}{4}(8a - 12b) + 16b}$

iv. $2.5m + 7 - 1.45m + 13$

v. $\frac{5}{8}(32p - 16q) + \frac{3}{7}(14 + 21q)$

vi. $36(\frac{3}{4}x + \frac{5}{6}y - \frac{3}{8}x) - \frac{4}{5}(10x + 20y)$

vii. $(6 + 3)w + 11w - 6$

4.2 Word statements into algebraic expressions

We simplify algebraic expressions to find the unknown numbers.

Steps

i. Identify the unknown quantity and give it a letter to represent it.

ii. Identify the operations to be used.

Example 4.

1. A father is five times the age of his daughter. If the sum of their ages is 42 years, find the age of the daughter.

Solution

Let the age of the daughter be y.

$$y + 5y = 42$$
$$6y = 42$$
$$y = \frac{42}{6} = 7$$

Example 5.

Maryanne has n mangoes, Winnie has half Maryanne's mangoes while Debborah has twice Maryanne's mangoes. If they have 28 mangoes all together, how many mangoes does each have?

Solution.

Let the number of maryanne's mangoes be n.

Winnie has $\frac{n}{2}$ mangoes

Debborah has $2n$ mangoes

$$n + \frac{n}{2} + 2n = 28$$
$$3n + \frac{n}{2} = 28$$

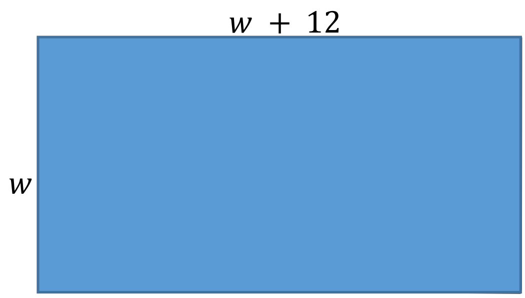
Multiplying both sides by 2

$$6n + n = 56$$
$$7n = 56$$
$$n = \frac{56}{7} \qquad n = 8$$

Example 6.

The length of a rectangle is greater than its width by 12cm. If the perimeter of the rectangle is 64cm, find its length and area.

Solution.



P=2(l+b)

Where $b=w$

$l=w+12$

Let the width be w.

Length = w + 12

$P = 2(l + l)$

$64 = 2\ (w + 12\ +\ w)$

$64\ = 2(2w\ +\ 12)$

$64\ =\ 4w\ +\ 48$

$4w\ =\ 64 - 48$

$4w\ =\ 16$

$w = \frac{16}{4}$ $\qquad w =\ 4\ cm$

Length $(l) =\ w\ +\ 12$

$=\ 4\ +\ 12 \qquad = \mathbf{16\ cm}$

Area $= l \times w$

$= 16 \times 4 \qquad = \mathbf{64cm^2}$

Activity 4:

1. A mother is 6 times older than her son. The sum of their ages is 63 years, determine the age of the mother.
2. The perimeter of a square is 36 cm. Find the length of its sides.
3. David's age divided by 3 is equal to David's age minus 14. Find David's age.

4.3 Evaluating expression by substitution

To solve an algebraic expression, we replace letters with numbers.

Example 7.

1. What is the value of 4p+5q?

When the value of p=3 and q= 2

$(4\times3) + (5\times2)$ Replace the letters with their number representation.

$$= 12 + 10$$

$$\underline{\underline{= \mathbf{22}}}$$

2. What is the value of? $(2p + 3y) - (2y + 3p)$

When the value of $p = 4$ and $y = 6$

$$(2\times p)+)\ (3\times y) - (2\times y) + (3\times p)$$

$$(2\times4) + (3\times6) - (2\times6) + (3\times4)$$

$$= (8 + 18) - (12 + 12)$$

$$= 26 - 24$$

$$\underline{\underline{= \mathbf{2}}}$$

Activity 5

In pairs what is the value of the expressions below:

When $p = 3, q = 5, r = 2, s = 7, c = 6, g = 4$

Show your working out.

a) $s + c - q$

b) $q\times g$

c) $q\times s \times p$

d) $p + c - r$

f) $c \times r$

g) $s + q - p$

h) $s \times c$

e) $q + s - g$

Example 8.

What is the value of?

$$\frac{1}{3}(3x+5y)+2y^2+\ 7p-\ 6$$

When

$$x=4,\ p=2x \text{ and } y=\frac{1}{2}x+\ 5$$

Therefore $x=\ 4,\ p=2\times 4=8$ and $y=\frac{1}{\cancel{2}}\times 4\overset{2}{\times} 6=12$

Solution

$$\frac{1}{3}(3\times 4+\ 5\times 12)\ +(2\times 12\times 12)+(7\times 8)-6$$

$$\frac{1}{\cancel{3}}\times \overset{4}{\cancel{12}}+\frac{1}{\cancel{3}}\times \overset{20}{\cancel{60}}+288+50$$

$$4+\ 20+\ 288+\ 50$$

$$=24+\ 338=\mathbf{362}$$

Exercise 1:

When done working out, check your answers with your partner.

1. Solve for p in the equation

   $2p + q + r = 10$ If $q = 4, r = 1$

2. Solve for w in the equation

   $x + w - z = 12$ If $x = 4, z = 2$

3. Given that $x = -2, y = 4$, determine the value of z in the equation

   $x + 2y - z = 0$

4. Given that $p = -3, q = -4$, determine the value ofr in the equation

   $3p - q = r$

5. If $x = -3, z = 10$, determine the value of y if the equation is

   $x + y = z$

6. When $c = 3, a = 4, b = 5$ what is the value of:

   a) $(c + a) - (b - a)$
   b) $(b + a) + (b - c)$
   c) $(b - c) + (a - c)$
   d) $(c \times a) - (b + a)$
   e) $(b \times a) + (c + b)$
   f) $(b - a) + (a - c)$

7. If h=8, g=7, f=5 what is the value of:

   a) $2(h - f)$
   b) $(h + h) - (g + f)$
   c) $3g + 5f - h$
   d) $(g - f) + (h - g)$

8. When $d = 4, e = 6, q = 2$ what is the value of:

   a) $(d \times q) + (e \times d)$
   b) $e + d - q$
   c) $3d + 2e - q$
   d) $(e + d) - (e - q)$
   e) $2(q + d)$
   f) $4q + 3d - 2e$

4.4 Sets

What is a Set?
A set is a well-defined collection of distinct objects.

Example 9.

A = {1, 2, 3, 4, 5}

What is an element of a Set?
The objects in a set are called its elements.
To learn about sets we shall use some accepted notations for the familiar sets of numbers.

Some of the different notations used in sets are:

Notation	Definition
$\in$	Belongs to
$\notin$	Does not belongs to
: or |	Such that
$\emptyset$	Null set or empty set
n(A)	Number of elements in set A
$\cup$	Union of two sets
$\cap$	Intersection of two sets
$\mathbb{N}$	Set of natural numbers = {1, 2, 3, ...}
$\mathbb{Z}_0^+$	Set of whole numbers = {0, 1, 2, 3, ...}
$\mathbb{Z}$	Set of integers = {........., -2, -1, 0, 1, 2, ...}
$\mathbb{Z}^+$	Set of all positive integers=$\mathbb{N}$
$\mathbb{Q}$	Set of all rational numbers
$\mathbb{Q}^+$	Set of all positive rational numbers
$\mathbb{R}$	Set of all real numbers
$\mathbb{R}^+$	Set of all positive real numbers
$\mathbb{C}$	Set of all complex numbers

These are the different notations in sets generally required while solving various types of problems on sets.

Note:

i. The pair of curly braces { } denotes a set. The elements of set are written inside a pair of curly braces separated by commas.
ii. The set is always represented by a capital letter such as; A, B, C....
iii. If the elements of the sets are alphabets then these elements are written in small letters.
iv. The elements of a set may be written in any order.
v. The elements of a set must not be repeated.
vi. The Greek letter Epsilon '$\in$' is used for the words 'belongs to', 'is an element of', etc.
vii. Therefore, $x \in A$ will be read as 'x belongs to set A' or 'x is an element of the set A'.
viii. The symbol '$\notin$' stands for 'does not belongs to' also for 'is not an element of'.

Therefore, $x \notin A$ will be read as 'x does not belong to set A' or 'x is not an element of the set A'.

Activity 6

From the notations learners should write the notations on cards and their explanations on different cards. Give them to their partners for them to match. Check if your partner got it correct.

4.5 Equivalent Sets

Two sets A and B are said to be equivalent if their cardinal number is same, i.e., $n(A) = n(B)$. The symbol for denoting an equivalent set is '$\leftrightarrow$'.

Example 10.

$A = \{1, 2, 3\}$ Here $n(A) = 3$
$B = \{p, q, r\}$ Here $n(B) = 3$
Therefore, $A \leftrightarrow B$

4.6 Equal sets

Two sets A and B are said to be equal if they contain the same elements. Every element of A is an element of B and every element of B is an element of A.

Example 11.

$A = \{p, q, r, s\}$
$B = \{p, s, r, q\}$

Therefore, $A = B$

Exercise 2:

Working in pairs, discuss which of the following pairs of sets are equivalent or equal.

a) $A = \{x : x \in N, x \leq 6\}$
   $B = \{x : x \in W, 1 \leq x \leq 6\}$
b) P = {The set of letters in the word ‘plane’}
   Q = {The set of letters in the word ‘plain’}
c) X = {The set of colors in the rainbow)
   Y = {The set of days in a week}
d) $M = \{4, 8, 12, 16\}$
   $N = \{8, 12, 4, 16\}$
e) $A = \{x \mid x \in N, x \leq 5\}$
   $B = \{x \mid x \in I, 5 < x \leq 10\}$

With your partner make a group with another pair;
One pair goes first and demonstrates using an example
How they know if a set is equivalent or equal
The other pair listens and then share their example.

So in case of the above Set A, the elements would be 1, 2, 3, 4, and 5. We can say, $1 \in A$, $2 \in A$

Usually we denote Sets by CAPITAL LETTERs like A, B, C, etc. while their elements are denoted in small letters like x, y, z

If x is an element of A, then we say x belongs to A and we represent it as $x \in A$

If x is not an element of A, then we say that x does not belong to A and we represent it as $x \notin A$

How to describe a Set?

Sets of Numbers

Natural Numbers ($\mathbb{N}$)

$\mathbb{N} = \{1, 2, 3,4 ,5\ 6, 7, ...\}$

Integers ($\mathbb{Z}$)

$\mathbb{Z} = \{..., -3, -2, -1, 0, 1, 2, 3, 4, ...\}$

Whole Numbers ($\mathbb{Z}_0^+$)

$\mathbb{Z}_0^+ = \{0, 1, 2, 3\ 4, 5, 6...\}$

Rational Numbers ($\mathbb{Q}$)

$\{\frac{p}{q}$: $p \in \mathbb{Z}$, $q \in \mathbb{Z}$, $q \neq 0\}$

Exercise 3.

List the elements of the following sets.

(a) A = The set of all even numbers less than 12

(b) B = The set of all prime numbers greater than 1 but less than 29

(c) C = The set of integers lying between -2 and 2

(d) D = The set of letters in the word LOYAL

(e) E = The set of vowels in the word CHOICE

(f) F = The set of all factors of 36

(g) $G = \{x : x \in N, 5 < x < 12\}$

(h) $H = \{x : x$ is a multiple of 3 and $x < 21\}$

(i) $I = \{x : x$ is perfect cube $27 < x < 216\}$

(j) $J = \{x : x = 5n - 3, n \in W$, and $n < 3\}$

(k) $M = \{x : x$ is a positive integer and $x2 < 40\}$

(l) N = {x : x is a positive integer and is a divisor of 18}
(m) P = {x : x is an integer and x + 1 = 1}
(n) Q = {x : x is a color in the rainbow}

4. Write each of the following sets.

(a) A = {5, 10, 15, 20}
(b) B = {l, 2, 3, 6, 9, 18}
(c) C = {P, R, I, N, C, A, L}
(d) D = {0}
(e) E = { }
(f) F = {0, 1, 2, 3, ..., 19}
(g) G = {—8, —6, —4, —2}
(h) H = {Jan, June, July}
(i) I = {a, e, i, 0, u}
(j) J = {a, b, c, d, ..., z}
(k) K = {1/1, 1/2, 1/3, 1/4, 1/5, 1/6}
(l) L = {1, 3, 5, 7, 9}

4.7 Finite Sets & Infinite Sets

Finite Set: A set where the process of counting the elements of the set would surely come to an end is called finite set.

Example 12.

All natural numbers less than 50
All factors of the number 36

Infinite Set: A set that consists of uncountable number of distinct elements is called infinite set.

Example 13.

Set containing all natural numbers $\{x : x \in \mathbb{N}, x > 100\}$

Cardinal number of Finite Set

The number of distinct elements contained in a finite set A is called the cardinal number of A and is denoted by n(A)

Example 14.

$A = \{1, 2, 3, 4\}$ then $n(A) = 4$
$A = \{x : x$ is a letter in the word 'APPLE'$\}$. Therefore $A = \{A, P, L, E\}$ and $n(A) = 4$
$A = \{x : x$ is the factor of $36\}$, Therefore $A = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$ and $n(A) = 9$

Empty Set

A set containing no elements at all is called an empty set or a null set or a void set.
It is denoted by ϕ
Also $n(\phi) = 0$

Example 15.

$\{x : x \in N, 3 < x < 4\} = \phi$

$\{x : x$ is an even prime number, $x > 5\} = \phi$

Non Empty Set

A set which has at least one element is called a non-empty set

Example 16.

$A = \{1, 2, 3\}$ or $B = \{1\}$

Equal Sets

Two set A and B are said to be equal sets and written as $A = B$ if every element of A is in B and every element of B is in A

Example 17.

$A = \{1, 2, 3, 4\}$ and $B = \{4, 2, 3, 1\}$

It is not about the number of elements. It is the elements themselves.

If the sets are not equal, then we write as $A \neq B$

Universal Set

If there are some sets in consideration, then there happens to be a set which is a super set of each one of the given sets. Such a set is known as universal set, to be denoted by U.

i.e. if $A = \{1, 2\}$, $B = \{3, 4\}$, and $C = \{1, 5\}$, then $U = \{1, 2, 3, 4, 5\}$

What is a Venn diagram?

A Venn diagram uses overlapping circles to illustrate the relationships between two or more sets of items. Often, they serve to graphically organize things, highlighting how the items are similar and different.

What are Venn diagrams?

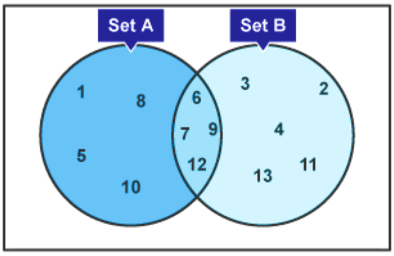
A Venn diagram is a way of grouping different items. These groups are known as sets.

We have a set of golf clubs or a set of dishes – these are just groups of those items.

We write a set using a special type of brackets. You could have a set of friends, eg {tom, lucy, marie}. Notice you **don't** use capitals within the brackets.

A Venn diagram begins with a box called our **universal set**, which is denoted by the symbol ε (epsilon).

The universal set contains everything we are interested in at that particular time. There'll be circles inside the box which we use to group the items within the universal set. Items in the circles form different subsets.



One Venn diagram can help solve the problem faster and save time. This is especially true when more than two categories are involved in the problem.

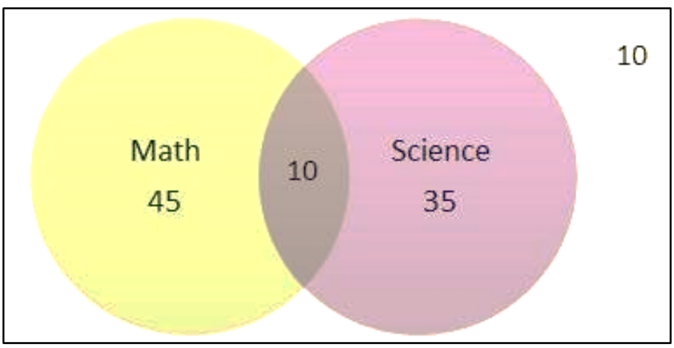
Example 18.

In a class of 100 learners, 35 like science and 45 like math. 10 like both. How many like either of them and how many like neither?

Solution:

Total number of learners, n = 100
Number of science learners, n(S) = 35
Number of math learners, n(M) = 45
Number of learners who like both, $n(M \cap S) = 10$
Number of learners who like either of them,
$n(M \cup S) = n(M) + n(S) - n(M \cap S)$
$\rightarrow 45+35-10 = 70$
Number of learners who like neither $= n - n(M \cup S) = 100 - 70 = 30$

The easiest way to solve problems on sets is by drawing Venn diagrams, as shown below.



Activity 7

In groups, use venn diagrams to solve the following;

1. There are 30 learners in a class. Among them, 8 learners are learning both English and Mathematics. A total of 18 learners are learning English. If every learner is learning at least one subject, how many learners are learning Mathematics in total?
2. In a group, there were 115 people whose proofs of identity were being verified. Some had passport, some had voter id and some had both. If 65 had passport and 30 had both, how many had voter id only and not passport?

   Among a group of people, 40% liked red, 30% liked blue and 30% liked green. 7% liked both red and green, 5% liked both red and blue, 10% liked both green and blue. If 86% of them liked at least one colour, what percentage of people liked all three?

Subsets

Set A is the numbers in the circle labelled Set A.
Set A = {1, 5, 6, 7, 8, 9, 10, 12}
Set B is the numbers in the circle labelled Set B.
Set B = {2, 3, 4, 6, 7, 9, 11, 12, 13}
These are the subsets of the universal.

Intersection

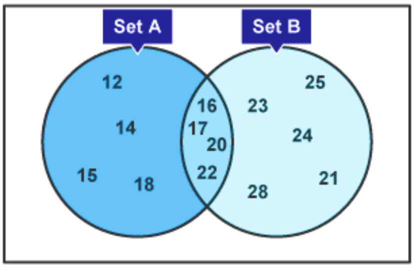
The intersection is where we have items from Set A and Set B, these can be found in the section that overlaps.
We write it as $A \cap B$. In the example above $A \cap B$ = {6, 7, 9, 12}.

Union

The union of a Venn diagram is the numbers that are in either Set A or Set B. The union of the above example is 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 as it's the numbers that appear in either of the circles.
We write it as $A \cup B$ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13}

Exercise 4.

1. Look at the venn diagram below.

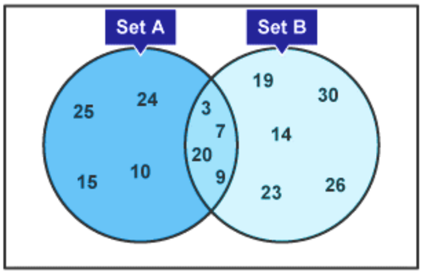


List the items in:

Set A

Set B

2. List the intersection and union of the following Venn diagram:



Activity 8:

Work in groups;

There are 150 Learners in primary 8 sitting some examination, if not all, of the following examinations: English, Maths and Science.

15 pupils are sitting both English and Maths but not Science

20 pupils are sitting Science and Maths but not English

18 pupils are sitting Science and English but not Maths

8 pupils are sitting all three exams

65 are sitting Science in total

55 are sitting English in total

72 are sitting Maths in total

Using a venn diagram, how many pupils did not sit any of these examinations?

UNIT 5: STATISTICS

5.1 Frequency Distribution

A frequency distribution is defined as an orderly arrangement of data classified according to the magnitude of the observations.

Frequency distribution helps us

1. To analyze the data.
2. To estimate the frequencies of the data.
3. To facilitate the preparation of various statistical measures.

Frequency and Frequency Tables

The **frequency** of a particular data value is the number of times the data value occurs.
A **frequency table** is constructed by arranging collected data values in ascending order of magnitude with their corresponding frequencies.

When the set of data values are spread out, it is difficult to set up a frequency table for every data value as there will be too many rows in the table. So we group the data into **class intervals** (or groups) to help us organize, interpret and analyze the data.
Each group starts at a data value that is a multiple of that group. For example, if the size of the group is 5, then the groups should start at 5, 10, 15, 20 etc. Likewise, if the size of the group is 10, then the groups should start at 10, 20, 30, 40 etc.

The **frequency of a group** (or class interval) is the number of data values that fall in the range specified by that group (or class interval).

Example 1.

The number of calls from motorists per day for roadside service was recorded for the month of December 2016. The results were as follows:

28	122	217	130	120	86	80	90	120	140
70	40	145	187	113	90	68	174	194	170
100	75	104	97	75	123	100	82	109	120
81									

Set up a frequency table for this set of data values.

Solution:

To construct a frequency table, we proceed as follows:

Smallest data value $= 28$

Highest data value $= 217$

$$\text{Difference} = \text{Highest value} - \text{Smallest value}$$
$$= 217 - 28$$
$$= 189$$

Let the width of the class interval be 40.

$$\therefore \text{ Number of class intervals} = \frac{189}{40} = 4.7 = 5 \qquad \{\text{Round up to the next integer}\}$$

There are at least 5 class intervals. This is reasonable for the given data.

Step 1: Construct a table with three columns, and then write the data groups or class intervals in the first column.

The size of each group is 40. So, the groups will start at 0, 40, 80, 120, 160 and 200 to include all of the data.

Note that in fact we need 6 groups (1 more than we first thought).

Class interval	Tally	Frequency
0 - 39		
40 - 79		
80 - 119		
120 - 159		
160 - 199		
200 - 239		

Step 2: Go through the list of data values. For the first data value in the list, 28, place a tally mark against the group 0-39 in the second column.

For the second data value in the list, 122, place a tally mark against the group 120-159 in the second column. For the third data value in the list, 217, place a tally mark against the group 200-239 in the second column.

Class interval	Tally	Frequency
0 - 39	I	
40 - 79		
80 - 119		
120 - 159	I	
160 - 199		
200 - 239	I	

We continue this process until all of the data values in the set are tallied.

Step 3: Count the number of tally marks for each group and write it in the third column. The finished frequency table is as follows:

Class interval	Tally	Frequency
0 - 39	I	1
40 - 79	~~IIII~~	5
80 - 119	~~IIII~~ ~~IIII~~ II	12
120 - 159	~~IIII~~ III	8
160 - 199	IIII	4
200 - 239	I	1
	Sum =	31

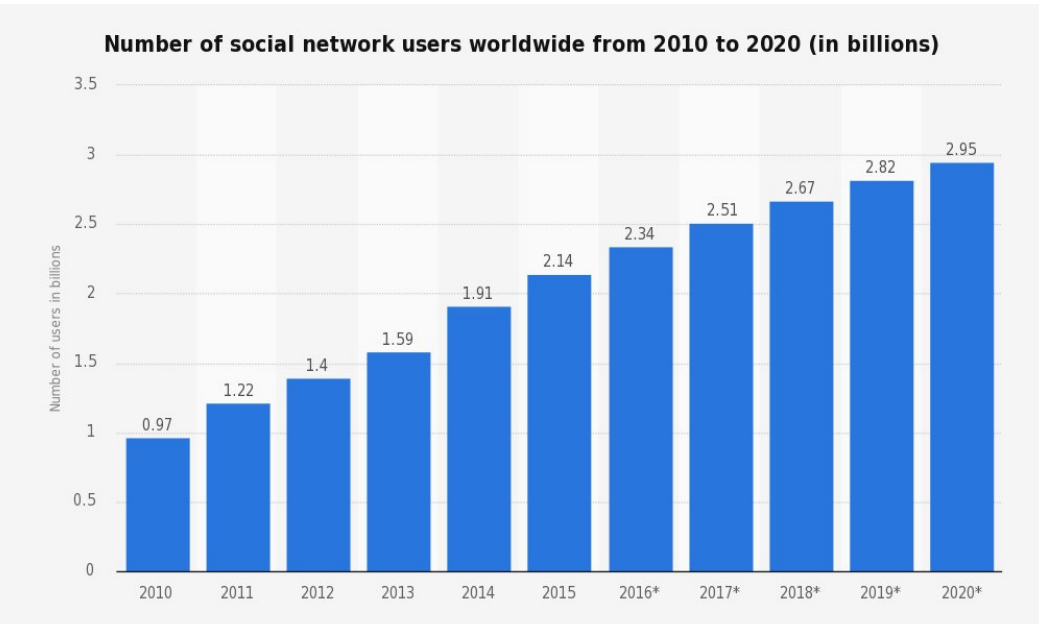
Exercise 1:

1. Construct the frequency distribution table for the data on heights (cm) of primary 7 pupils using the class intervals 130 - 135, 135 - 140 and so on. The heights in cm are: 140, 138, 133, 148, 160, 153, 131, 146, 134, 136, 149, 141, 155, 149, 165, 142, 144, 147, 138, 139.
2. Construct a frequency distribution table for the following weights (in gm) of 30 oranges using the equal class intervals, one of them is 40-45 (45 not included). The weights are: 31, 41, 46, 33, 44, 51, 56, 63, 71, 71, 62, 63, 54, 53, 51, 43, 36, 38, 54, 56, 66, 71, 74, 75, 46, 47, 59, 60, 61, 63.

Activity 1

Measure the heights of all the learners in the class and record the heights. Construct a frequency distribution table.

Look at the graph below.



In pairs, discuss what you can see in the graph. Do you see any difference in the year 2010 and 2020.

5.2 Data collection Process

Step 1: Identify issues for collecting data

The first step is to identify issues for collecting data and to decide what next steps to take.

To do this, it may be helpful to conduct a quick assessment to understand what is happening around the area you want to collect data from.

Step 2: Select issue(s) and set goals

The focus of Step 2 is choosing a priority issue(s) for collecting data, and then setting goals and objectives.

Select some of the questions to consider when deciding to prioritize an issue for gathering data.

Step 3: Plan an approach and methods

In Step 3, we make decisions about how data will be collected, the sources of data that will be used, and the duration of the data collection will take, among other questions.

The methods and approaches will flow from the goals set in Step 2, and will vary significantly depending on the purpose and complexity of the issue(s) selected.

Step 4: Collect data

When planning on how best to collect data in Step 4, it is important to be aware of the practical considerations and best practices for addressing logistical challenges people often face at this stage. Like transport and communication

Step 5: Analyze and interpret data

Step 5 involves analyzing and interpreting the data collected. Whether quantitative and/or qualitative methods of gathering data are used, the analysis can be complex, depending on the methods used and the amount of data collected.

Step 6: Act on results

Once we analyze and interprete the results of the data collected, we can decide to act on the data, collect more of the same type of data or modify its approach.

5.3 Reading and interpreting tables and graphs

This is the fifth step where we already have the data collected.

Example 2.

The table below shows the class attendance in a school. All pupils were present on Friday. Which day had the highest number of absentees?

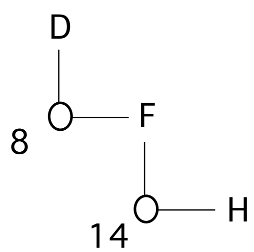
DAYS	PRESENT
Monday	48
Tuesday	49
Wednesday	47
Thursday	48
Friday	50

∴ The highest number of absentees was on Wednesday

Example 3.

Mercy and Juma travelled from D to H via F. How many kilometres did they travel if they used the table below?

D				
80	**E**			
88	70	**F**		
100	90	60	**G**	
200	160	140	70	**H**



$$88 + 140 = 228km$$

Activity 2:

1. The table below shows the fare in South Sudanese Pounds for a bus travelling to different towns.

A					
50	**B**				
80	60	**C**			
100	80	40	**D**		
150	100	120	50	**E**	
200	150	130	90	60	**F**

A teacher and 3 pupils left town A for town F. They stopped at town C and then continued with the journey to town F in another bus. If the bus fare for children is half that of adults, how much did they pay altogether?

Exercise 2.

1. The table below shows the number of times football teams Team A, Team B and Team C won, drew or lost in a competition. Three points were awarded for each game won, one point for each game drawn and no point for a game lost.

	Team A	**Team B**	**Team C**
WON	2	3	4
DRAWN	2	4	3
LOST	5	2	5

Arrange in order the 1st, 2nd and 3rd team in the score. Which two teams tied?

2. The table below shows the number of tonnes of sugar produced and sold by a factory in 7 days.

Days	**MON**	**TUE**	**WED**	**THUR**	**FRI**	**SAT**	**SUN**
Tonnes produced	500	700	800	400	900	250	250
Tonnes sold	300	400	200	700	500	150	180

On which day was the number of tonnes of sugar sold one and three quarter times the number of tonnes produced?

3. A trader sold loaves of bread for all the days of the week. The table below shows the number of loaves the trader sold in 6 days of the week.

DAYS	**MON**	**TUE**	**WED**	**THUR**	**FRI**	**SAT**	**SUN**
No. of loaves of bread	150	95	105	80	40	91	70

One loaf of bread was sold at SSP100. How much did the trader get on Monday than on Saturday?

4. The table below shows the number of people who attended an agricultural show.

Female Adults	**Male Adults**	**Children**
1909	3918	3449

How many more adults than children attended the show?

5. The table below represents arrival and departure times of buses from a company serving towns J, K, L, M, N, P, Q and R.

Towns	**Arrival Time**	**Departure Time**
J		6.00 am
K	8.30 am	9.30 am
L	10.20 am	10.30 am
M	11.45 am	12.00 noon
N	12.45 pm	1.00 pm
P	2.05 pm	2.20 pm
Q	3.15 pm	3.30 pm
R	4.45 pm	

How long does it take the bus to travel from town K to town Q?

6. The table below shows the number of pupils who were present from Monday to Friday.

DAYS	**MON**	**TUE**	**WED**	**THUR**	**FRI**
No. of pupils	63	57	65	67	69

If the class has a total of 70 pupils. How many pupils were absent on Tuesday?

7. The table below represents the sales of milk in litres by a milkman in five days. The sale for Friday is not shown.

DAYS	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
NUMBER OF LITRES	24	20	20	25	_____

One litre of milk was sold at SSP 50. The milkman got a total of SSP 5,850 for the sale of milk during the five days. How many more litres of milk did the milkman sell on Friday than on Tuesday?

8. The table below shows the number of pupils who were in standard 5–8 in a certain school from 2011- 2014.

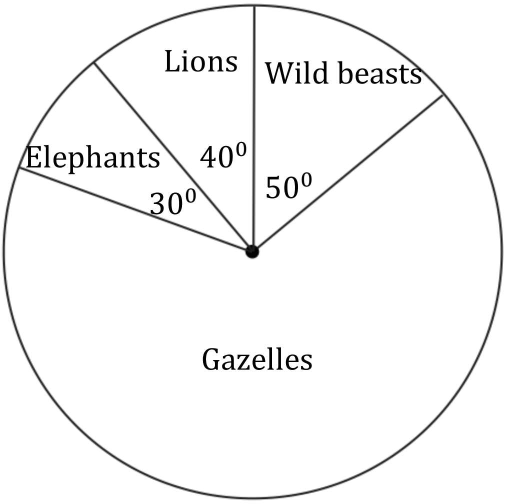
	Primary 5	**Primary 6**	**Primary 7**	**Primary 8**
2011	81	75	61	57
2012	85	79	73	59
2013	88	82	76	70
2014	91	85	79	73

How many pupils of the class which was in Primary 5 in 2011 had dropped out of that class by 2014?

Working out problems involving pie charts

Example 4.

The pie chart below shows the population of 10800 wild animals in a certain part.



How many more Gazelles than wild beasts are there in the park?

Solution

A circle has 360˚ which represent 10800 animals

$1^\circ = \frac{10800}{360^\circ}$

Angle representing gazelles is 360 - (30˚+ 40˚+ 50˚)

$= 360^\circ - 120^\circ$

$= 240^\circ$

$1^\circ = \frac{10800}{360^\circ}$

$240^\circ = \frac{\cancel{10800}}{\cancel{360^\circ}}30 \times 240^\circ \quad = 7200$ Gazelles

$1^\circ = \frac{\cancel{10800}}{\cancel{360^\circ}}30 \times 50^0 \quad = 1500$ Wild beasts

How many more gazelles than wild beasts?

7200 - 1500= 5700

There are 5700 more gazelles than wild beasts in the park.

Activity 3:

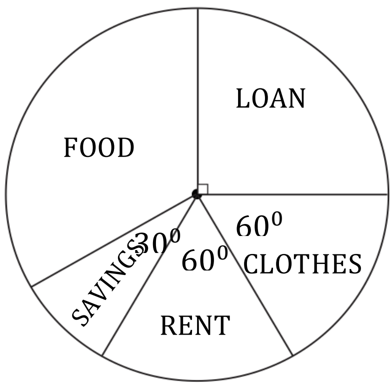
In groups, work out the activities and present to the class.

1. The table below represents different colours and the number of pupils who like each colour.

Colours	**Red**	**Blue**	**White**	**Yellow**	**Green**
Number of pupils	14	16	12	26	22

Draw a pie chart to represent the information given above, what will be the difference in angles of pupils who liked blue colour compared to those who like colour green?

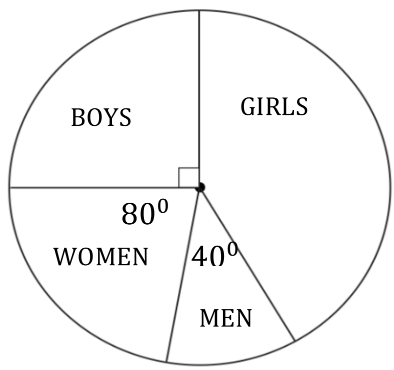
2. The pie-chart below shows how Taban spent his salary.



How much more did he spend on loan than on rent, if he spent SSP8000 on food?

3. The population of an estate in a town is represented by the pie chart below.

If there were 600 girls, how many more boys than men were there?



Exercise 3.

1. The table below shows the number of exercise books each pupil was given

Exercise books	Nene	Maundu	Ann	Mustafa	Asha
Number of pupils	16	14	15	18	27

Draw a pie chart to represent this information.

2. There were 210 girls, 168 boys, 336 men and 126 women in a meeting. If a pie chart was drawn to represent this information. What angle would represent the boys?

3. The table below shows Kenyi's score in topical tests in mathematics. The tests were marked out of 20 marks.

Tests	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}
Score	18	17	20	16	15	19	15

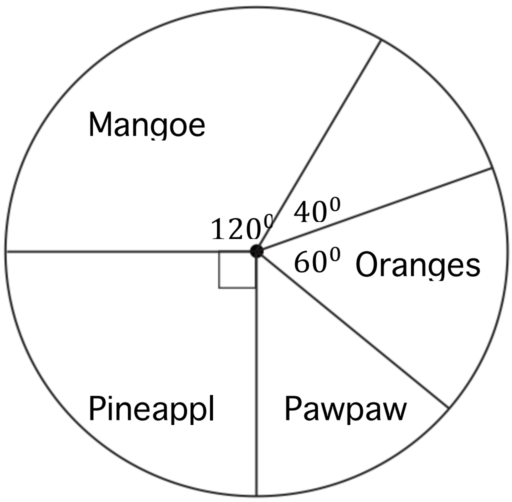
If a pie chart is to be drawn to represent the information given in the table above, what would be the sum of angles representing 2^{nd} and 5^{th} tests?

4. The table below shows Nyandeng's income from the sale of farm produce during a certain year. The information on the income of vegetables is not given.

Produce	**Cabbages**	**Coffee**	**Maize**	**Wheat**
Income	________	SSP 72 000	SSP 42 000	SSP 90 000

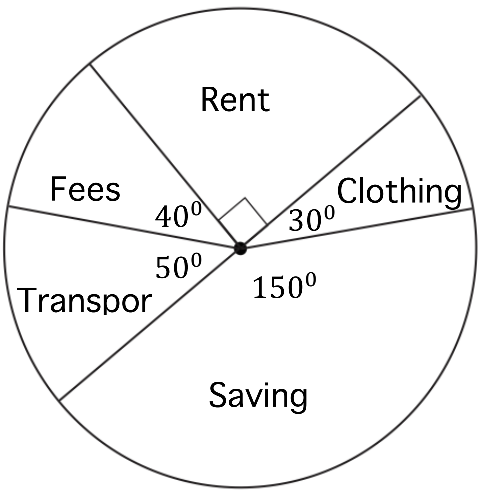
In groups, draw pie chart was drawn to represent the information above. If the angle representing the income from maize was 72°, what was the income from cabbages?

5. The circle graph below shows 432 fruits sold by a trader in one day.



How many more oranges than pawpaw did the seller sell that day?

6. The pie chart below shows how Eunice spent her June salary.



If she spends SSP 2,400 on clothing every month, how much less does she spend on transport than on rent?

Activity 4:

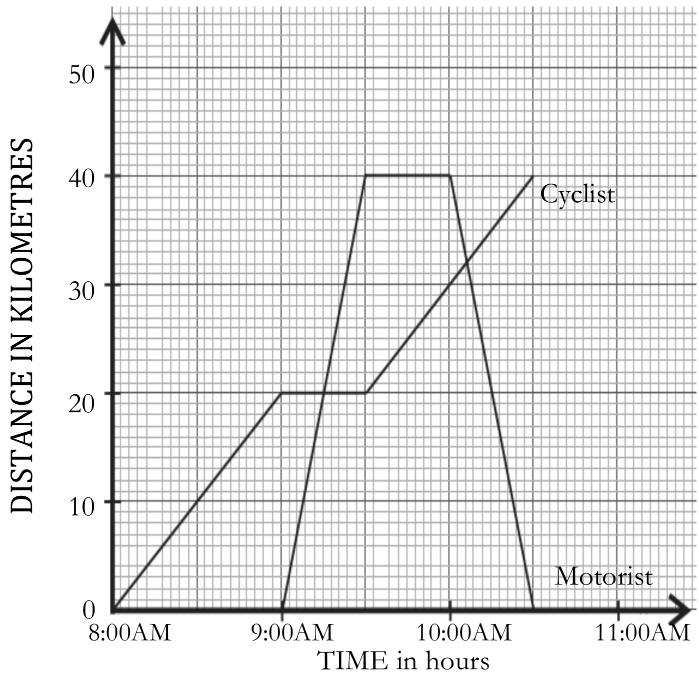
2. The table below shows how Akello utilizes his piece of land.

Purpose	Homestead	Tea cultivation	Maize cultivation	Grazing
Number of hectares	$\frac{1}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$

Using a graph paper, plot the data on a bar graph.

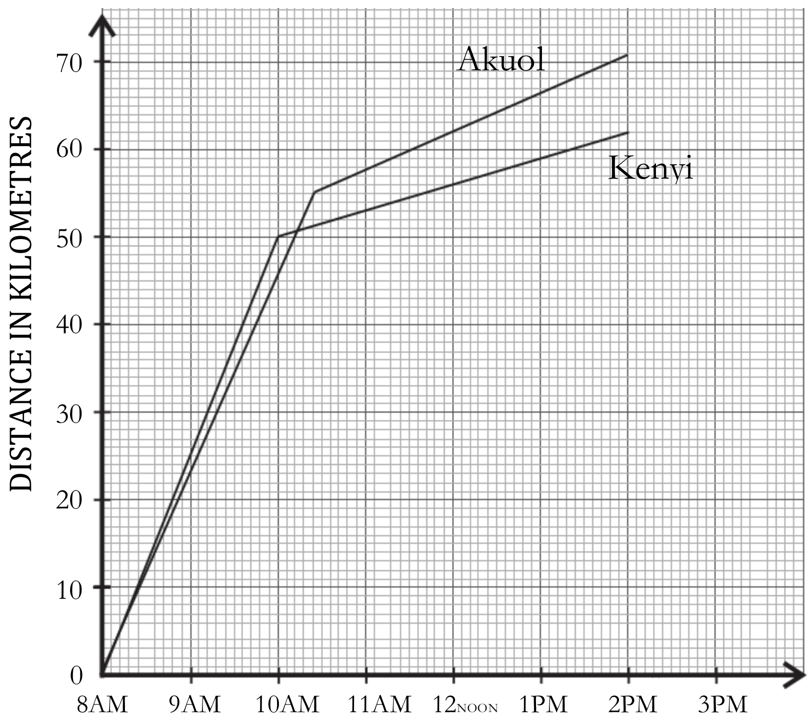
Exercise 4.

1. Below is a travel graph showing the journey of a motorist travelling from town R to town S and back, and that of a cyclist travelling from town R to town S.



How far from R was the cyclist when she met the motorist travelling back to town R?

2. The graph below shows journeys of two motorists, Akuol and Kenyi.



How far away from starting point did the two motorists meet?
How far had Kenyi travelled at 10.00AM?

5.4 The mean

To calculate the mean, simply add all of your numbers together.
Next, divide the sum by however many numbers you added. The result is your *mean* or average score.

Example 5.

Let's say you have four test scores: 15, 18, 22, and 20. To find the average, you would first add all four scores together, then divide the sum by four. The resulting mean is 18.75. Written out, it looks something like this:
$(15 + 18 + 22 + 20) / 4 = 75 / 4 = 18.75$

Exercise 5:

Calculate the mean of the following groups of data.

a. 97, 11, 13, 21, 70, 61, 45, 85, 87

b. 5, 38, 79, 5, 2, 50, 69, 16, 70, 27

c. 76, 13, 22, 74, 20, 1, 1, 74, 10

d. 32, 50, 78, 69, 50, 46, 22, 76, 94

e. 60, 17, 11, 70, 18, 25, 70, 90, 17

5.5 The median

The median is the middle value in a data set.

To calculate it, place all of your numbers in increasing order. If you have an odd number of integers, the next step is to find the middle number on your list.

Example 6.

Find the median.

3, 9, 15, 17, 44

The middle or median number is 115

If you have an even number of data points, calculating the median requires another step or two.

First, find the two middle integers in your list. Add them together, then divide by two.

The result is the median number.

Example 7.

Find the median.

3, 6, 8, 12, 17, 44

The two middle numbers are 8 and 12.

Written out, the calculation would look like this:

$(8 + 12) \div 2 = \frac{20}{2} = 10$

In this instance, the median is 10.

Exercise 6:

Find the mean and median for the following list of values:

13, 18, 13, 14, 13, 16, 14, 21, 13

5.6 The mode

The mode is about the frequency of occurrence. There can be more than one mode or no mode at all; it all depends on the data set itself. For example, let's say you have the following list of numbers:

3, 3, 8, 9, 15, 15, 15, 17, 17, 27, 40, 44, 44

In this case, the mode is 15 because it is the integer that appears most often. However, if there were one fewer 15 in your list, then you would have four modes: 3, 15, 17, and 44.

Exercise 7:

Calculate the Median for Each of the Sets of Numbers:

1.18, 38, 46, 7, 12, 43, 11

2. 23, 48, 6, 1, 3, 8, 1

3. 34, 50, 20, 44, 30, 49

4. 34, 26, 30, 18, 7, 30

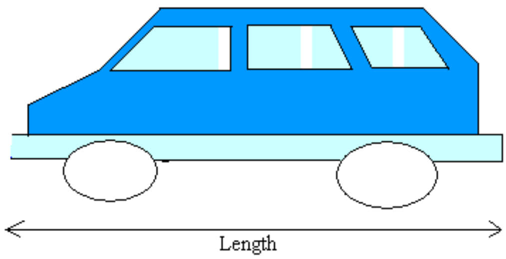
5. 30, 7, 9, 36, 32, 44, 29

Activity 5

Using the data collected in activity 1, calculate the mean, median and mode.

5.7 Scale Drawings

Since it is not always possible to draw on paper the actual size of real-life objects such as the real size of a car, an airplane, we need scale drawings to represent the size like the one you see below of a van.



In real-life, the length of this van may measure 240 inches. However, the length of a copy or print paper that you could use to draw this van is a little bit less than 12 inches

Since $\frac{240}{12} = 20$, you will need about 20 sheets of copy paper to draw the length of the actual size of the van

In order to use just one sheet, you could then use 1 inch on your drawing to represent 20 inches on the real-life object

You can write this situation as 1:20 or $\frac{1}{20}$ or 1 to 20

Example 8.

The length of a vehicle is drawn to scale. The scale of the drawing is 1:20

If the length of the drawing of the vehicle on paper is 12 inches, how long is the vehicle in real life?

Set up a proportion that will look like this:

$$\frac{\text{Length of drawing}}{\text{Real Length}} = \frac{1}{20}$$

Do a cross product by multiplying the numerator of one fraction by the denominator of the other fraction

Length of drawing × 20 = Real length × 1

Since length of drawing = 12, we get:

$$\begin{aligned} 12 \times 20 &= Real\ length \times 1 \\ 240\ inches &= Real\ length \end{aligned}$$

Exercise 8:

Show your work out.

1. A map has a scale of 1cm: 3 miles. On the map, the distance between two towns is 7cm. What is the actual distance between the two towns

2. The diagram shows part of a map. It shows the position of a school and a shop.

The scale of the map is 1cm = 100 metres.

Work out the real distance between the school and the shop. Give your answer in metres.

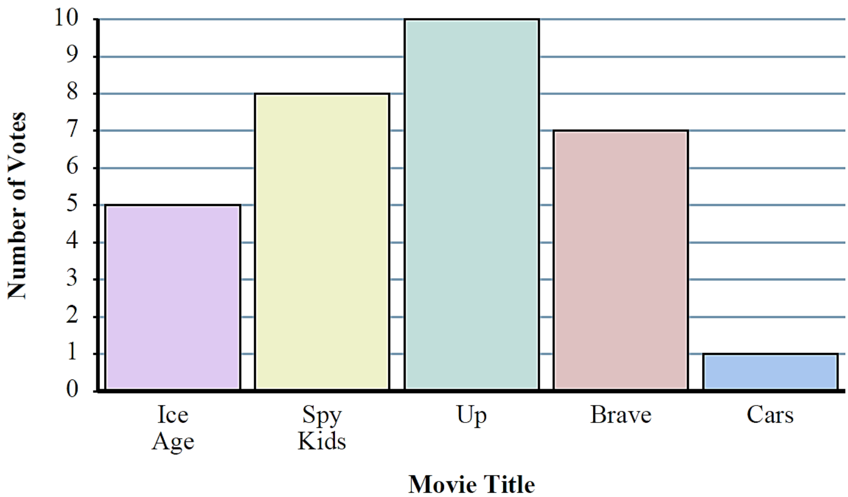
3. A map has a scale of 1cm: 4 kilometres. The actual distance between two cities is 52 kilometres. What is the distance between the cities on the map?
4. A map has a scale of 1:4000. On the map, the distance between two houses is 9cm. What is the actual distance between the houses? Give your answer in metres.
5. A scale drawing has a scale of 1:20. In real life the length of a boat is 150m. What is the length of the boat on the scale drawing? Give your answer in centimetres.

5.8 Graph

Bar Graph

A Bar Graph (also called Bar Chart) is a graphical display of data using bars of different heights.

At home the learners had to vote on which movie to watch. The voting results are listed below. Use the bar graph to answer the questions.



Work in pairs;

1) How many people voted for Ice Age?
2) Did more people vote for Ice Age or for Up?
3) Did fewer learners vote for Cars or for Brave?
4) Which movie received exactly 10 votes?
5) What is the difference in the number of people who voted for Brave and the number who voted for Spy Kids?
6) What is the combined number of people who voted for Up and Brave?

Circle Graphs or Pie Charts

A pie chart (also called a Pie Graph or Circle Graph) makes use of sectors in a circle. The angle of a sector is proportional to the frequency of the data.

The formula to determine the angle of a sector in a circle graph is:

$$\text{Angle of sector} = \frac{\text{Frequency of data}}{\text{Total frequency}} \times 360°$$

Study the following steps of constructing a circle graph or pie chart:

Step 1: Calculate the angle of each sector, using the formula

$$\text{Angle of sector} = \frac{\text{Frequency of data}}{\text{Total frequency}} \times 360°$$

Step 2: Draw a circle using a pair of compasses
Step 3: Use a protractor to draw the angle for each sector.
Step 4: Label the circle graph and all its sectors.

Example 9.

In a school, there are 750 learners in Year1, 420 learners in Year 2 and 630 learners in Year 3. Draw a circle graph to represent the numbers of learners in these groups.

Solution:

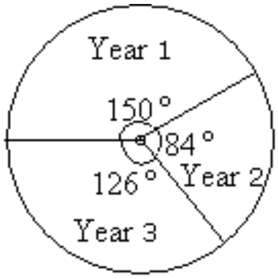
Total number of learners = 750 + 420 + 630 = 1,800.

$$\text{Year 1: size of angle} = \frac{750}{1800} \times 360° = 150°$$

$$\text{Year 2: size of angle} = \frac{420}{1800} \times 360° = 84°$$

$$\text{Year 3: size of angle} = \frac{630}{1800} \times 360° = 126°$$

Draw the circle, measure in each sector. Label each sector and the pie chart.



Groups of students in a school

Activity 6

Look at this record of traffic travelling down a particular road.

Type of vehicle	Number of vehicles
Cars	140
Motorbikes	70
Vans	55
Buses	5
Total vehicles	270

Drawing a pie chart.

5.9 Probability

Probability is the measure of the likelihood that an event will occur.

To show random events have different likelihood of occurring.

Use vocabulary of chance e.g. impossible, certain, equally likely, even chance, unlikely, likely etc. to answer the questions.

An event that is certain to happen has a probability of 1.

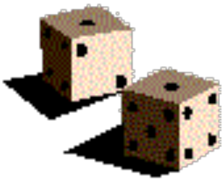
An event that cannot possibly happen has a probability of zero.

If there is a chance that an event will happen, then its probability is between zero and 1.

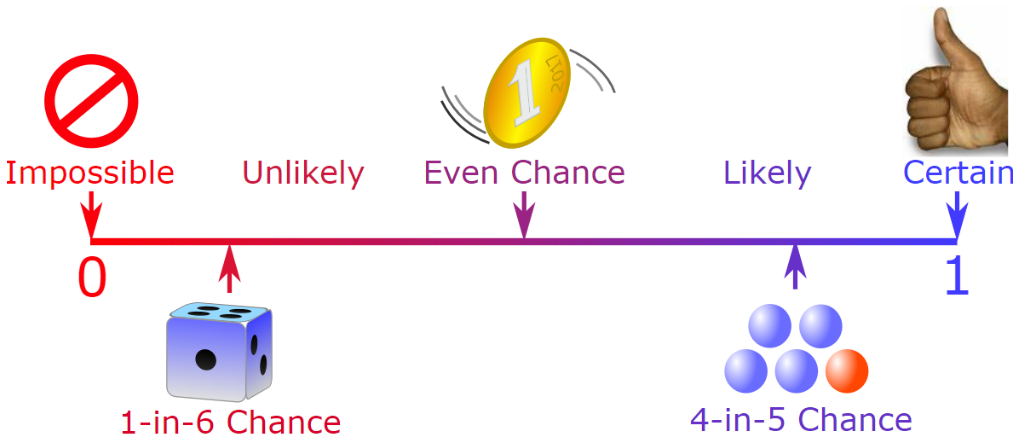
Examples of Events:

- Tossing a coin and it landing on *heads.*
- Tossing a coin and it landing on *tails.*
- Rolling a '3' on a die.

- Rolling a number > 4 on a die.
- It rains two days in a row.
- Drawing a card from the suit of clubs.
- Guessing a certain number between 000 and 999 (lottery).



Create a set of statements in your group that another group has to agree or disagree with.



Events that are certain:

- If it is Thursday, the probability that tomorrow is Friday is certain, therefore the probability is 1.
- If you are sixteen, the probability of you turning seventeen on your next birthday is 1. This is a certain event.

Events that are uncertain:

- The probability that tomorrow is Friday if today is Monday is 0.
- The probability that you will be seventeen on your next birthday, if you were just born is 0.

Let's take a closer look at tossing the coin. When you toss a coin, there are two possible outcomes, "heads" or "tails."

Examples of outcomes:

- When rolling a die for a board game, the outcomes possible are 1, 2, 3, 4, 5, and 6.
- The outcomes when choosing the days of a week are Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, and Saturday.

Activity 7

In groups, collect different marbles or any available safe materials to do the activity.

Materials: Sack; marbles of two different colors - 100 of one color (blue), 25 of another color (green).

Procedure:

Put all the marbles in the sack.

We will try to find out - without looking in the sack and counting whether there are more blue marbles or more green marbles in the sack.

Have four learners draw five marbles each from the sack. (Make sure that the marbles are put back into the sack after each draw.)

Have every learner record the numbers and colors of marbles for each of the four draws.

Questions

1. On the basis of the first four draws how many marbles of each color are there in the sack?
   Let each learner in the rest of the class draw five marbles each from the sack. (Be sure to put the marbles back in the sack after each drawing.)
2. What are the totals for each color of marble?
3. Do you think there were more marbles of one color than the other? Why?
4. If so, what do you think the ratio of one color to the other might be? G. Open the sack and count the number of marbles of each color.
5. What is the ratio of one color to the other color?

Example 10.

If you draw a card from a standard deck of cards, what is the probability of not drawing a spade?

Solution

There are 13 spades, so that means that there are 52 – 13 = 39 cards that are not spades.

$$\frac{39}{52} = \frac{3}{4} = 75\%$$

Example 11.

If you roll two dice, what is the probability that the sum of the two is odd?

Solution

there are 18 combinations that result in an odd sum. There are still 36 different combinations, so:

$$\frac{18}{36} = \frac{1}{2} = 50\%$$

Example 12.

Jermain and Tremain both calculate the predicted probability of getting heads if they flip a coin 10 times. Then they each flip a coin 10 times.

a. Will they get the same number when they calculate the predicted probability?
b. When they actually flip the coin 10 times, will they get as many as the probability predicted, for sure, no matter what?
c. When they actually flip the coin 10 times, will Jermain absolutely, positively get the same number of heads as the Tremain?

Solution:

a. Since they're using the same formula to find the predicted probability, they will get the same number:
b. The number of heads they flip is all up to chance. They should flip about 5 heads out of their 10 flips, but there's no promise, no absolute way of knowing. The answer is no, there is no guarantee that they'll flip 5 heads.

c. Because what they actually flip is up to chance, there is also no guarantee that Jermain will flip the same number of heads and the same number of tails as Tremain. It's just flippin' up to chance.

Activity 8:

In pairs toss a coin 20times and record the outcome in a table.

Heads		
Tails		

Record the class result

1. What are the chances of getting a head?
2. What are the chances of getting a tail?
3. Does the coin know what had happened on the last throw?
4. Is it more or less likely to get a head or a tail?
5. Is getting an even score on a head as likely as getting a tail an odd score?

Activity 9:

In pairs roll a die 48 times and record the outcome e.g. 5, 4, 3, 3etc. Count the total number of each score and make a table and a bar chart.

Create a table of the class results and make a bar chart. Using a bar graph answer these questions

a. What are the chances of getting a particular score?
b. Is it possible to have a draw in this game?
c. Is getting an even score on a dice as likely as getting an odd score?
d. Is each outcome equally likely?
e. Is this game fair or unfair? Explain to the rest of your classmates.

Activity 10:

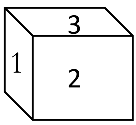
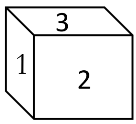
In pairs toss a bottle top 20 times and record the outcome in a table. Fill in the data in a table.

a. Does the bottle top behave in the same way as the coin? If not, why?
b. Is it possible to have a draw in the outcome?
c. What are the chances of getting a particular score?
d. Make a list of possible outcomes?

Activity 11:

In pairs make 2 cubes and number their faces 1- 6.

Roll two number cubes, if the product is odd, player x wins. If the product is even player y wins. Play 20 times of this game. Record the result in a table.

Player X	**Player Y**
	

a. How many different outcomes are possible?
b. How likely is the product even?
c. Is it equally likely to have a draw in in this game?
d. How many ways can player y win?
e. How many ways can player x win?
f. Is this game fair or unfair?

Exercise 9.

Find the probability. Write your answer as a fraction in the simplest form:

1. There are 6 green marbles and 2 red marbles in a jar. What the probability of picking a red marble.
2. There are 5 lollipops and 4 candies marbles in a Jar. What is the probability of picking a lollipop?

3. There are 6 maize grains and 2 beans in a bag. What is the probability of picking a bean?
4. There are 10 black and 9 white crayons in a box. What is the probability of picking a white crayon?
5. A glass jar contains 15 red and 11 blue marbles. What is the probability of picking a blue marble?

UNIT 6: BUSINESS ACCOUNTING

Look at the picture. In pairs, discuss what you can see.

6.1 Commissions in South Sudanese Pounds

Commission is the money given to a sales person after the sale of goods on behalf of the employer.

To calculate the commission in South Sudanese Pounds we multiply R% by value of goods sold.

Example 1.

Deng sold goods worth SSP 25 000 for a certain company. He was given a commission of 15% of the value of the goods sold. How much was his commission?

Solution

Value of goods sold = SSP 25,000

Percentage commission rate = 15%

Commission in South Sudanese Pounds = R%×Value of goods sold

=15/100×25,000

=SSP 3,750

Exercise 1.

1. Nadia was paid a salary of SSP 25 200 per month. She is also paid a commission of 9% on the value of goods sold. In one month she sold goods worth SSP150 000. What was the total money she got at the end of that month?
2. Michael was paid a salary of SSP 27 000 per month and commission at $6\frac{1}{2}\%$ for the value of the goods sold. Above SSP10 000. In June he sold goods worth SSP140 000, what was his total money he received at the end of the month?
3. George is paid a commission of 12.5% on the value of goods sold above SSP 25 000. In one month she sold goods valued at SSP175,000. What was her commission that month?
4. A salesperson is paid a commission on the value of goods sold. In one month she sold 600 items each at SSP1 500. If she was paid a commission at the rate of 5%, how much commission in South Sudanese Pounds did she earn?
5. Khamis is paid a monthly salary of SSP 30,000 and a 3% commission on goods he sold above SSP150 000. In December he sold goods worth SSP125 000, how much was his total earnings at the end of that month?

6.2 Discounts in South Sudanese Pounds

Discount involves reducing the prices of items to attract customers into buying them.

Example 2.

Angelo bought a bed whose marked price was SSP15,000. If he bought it for SSP13,910, what discount was he allowed for the bed?

Solution

$Marked\ price\ = SSP15{,}000$

$Selling\ price\ = SSP13{,}910$

$$Discount\ = SSP15{,}000 - SSP13{,}910\ = SSP1{,}090$$

$$Discount\ = Marked\ price - Selling\ price$$

Activity 1:

1. Pricilla bought a dress for SSP650, the marked price was SSP810. How much discount was Pricilla given?
2. Adil paid SSP2 750.50 for a wardrobe. If the marked price of the wardrobe was SSP3 600, how much was the discount?
3. A customer bought an item for SSP 750, after he was given a discount of SSP150. What was the marked price of the item?
4. Solomon paid SSP8 500 for a radio after getting a discount of SSP95. How much less would he have paid had he been given a discount of SSP115?

6.3 Hire purchase in South Sudanese Pounds

This is a method of buying items over a period of time. Deposit is the amount of money paid first. Instalment is the amount paid thereafter over the given period of time.

$$Hire\ purchase\ =\ Deposit + Total\ instalment$$

Example 3.

The hire purchase of a bicycle is SSP 7 500. Francis paid SSP 1 000 as deposit and the balance was paid in 10 equal monthly instalments. How much was each monthly instalment?

Hire purchase = SSP 7,500
Deposit = SSP 1,000

$$\text{Total instalment} = \text{Hire purchase} - \text{Deposit}$$

$$\text{Total instalment} = 7500 - 1000$$

$$\text{Total instalment} = 6500$$

Monthly instalment = $6500 \div 10$ = SSP 650

Exercise 2.

1. The hire purchase price of a sewing machine was 125% of the cash price. Luka bought the sewing machine at hire purchase terms by paying a deposit of SSP13 500 plus 9 months installments of SSP1,500. What was the cash price of the sewing machine?
2. Noah bought a radio cassette on hire purchase price. He paid a deposit of SSP 45 000 and 18 equal monthly installments of SSP 850. The total amount paid was 25% more than the cash price. What was the price of the radio cassette?
3. Gabriel bought a laptop on hire purchase terms. He paid a deposit of SSP 20 000. The remaining amount was paid in 8 equal monthly installments. He paid a total of SSP27680. How much was each monthly installment?
4. The hire purchase price of a dining table is 120% of the marked price. The hire purchase price is a deposit of SSP 8 000 and an instalment of SSP 5 000 each. By how much is the hire purchase more than the marked price?
5. Philip bought a TV set on hire purchase items. She paid a deposit of SSP120 000 and 12 equal monthly installments of SSP 850 each. The hire purchase price was 20% more than the cash price. Sylvia bought the same TV set on cash. How much more did Alai pay for the TV set?
6. The hire purchase price of an electric cooker was 10% more than the cash price. The cash was SSP150 000. Antony paid SSP 90,000 as deposit and the rest in equal monthly instalments for 18 months. How much was his monthly instalment?
7. The marked price of a motorcycle was SSP 300 000 but a discount of 8% was allowed for cash payment. Ryan bought the motorcycle on hire purchase by paying a deposit of SSP12 000 followed by 8 equal monthly instalment of SSP18 000 each. How much money would Ryan have saved had he bought it for cash?

6.4 Profit and loss in South Sudanese Pounds

Profit is realized when the selling price is higher than the buying price.

Example 4.

Lopir bought a basin for SSP175. He later sold it for SSP208. What profit did Lopir make?

Solution

Selling price	=	SSP208
Buying price	=	SSP172
Profit	=	SSP (208-172)
	=	SSP36

$$Profit = Selling\ price - Buying\ price$$

Loss is realized when the buying price is higher than the selling price.

Example 5.

Worija bought a radio for SSP 720. He later sold it at SSP 630. What loss did Worija make?

Solution

Buying price	=	SSP720
Selling price	=	SSP630
Loss	=	SSP(720-630)
	=	SSP90

$$Loss = Buying\ price - Selling\ price$$

Exercise 3.

1. A trader bought 8 trays of eggs at SSP 240 per tray, eight eggs broke and he sold the rest at SSP8 per egg. If a tray holds 30 eggs, how much loss did he get?
2. Jacob bought 250 chicken whose average mass was $1\frac{1}{2}$kg. The buying price per kilogram was SSP150. He then sold each chicken for SSP 215, what profit did Jacob make?

3. Jacinta bought 15 bags of fruits at SSP 450 per bag. She spent SSP 500 on transport, $1\frac{1}{2}$ bags of the fruits got spoilt and she sold the rest at SSP 400 per bag. What was her loss?
4. Saida bought 9 trays of eggs @ SSP 200. All eggs in one of the trays broke and he sold the remaining trays @ SSP 205. What loss did he make?

6.5 Simple interest in South Sudanese Pounds

This type of interest usually applies to automobile loans or short-term loans, although some mortgages use this calculation method.
Terms used in Simple Interest and Compound Interest:
Principal: This is the money borrowed or lent out for a certain period of time is called the principal or sum.
Interest: Interest is payment from a borrower to a lender of an amount above repayment of the principal sum.
Amount: The total money paid back by the borrower to the lender is called the amount.

Amount = Principal + Interest

Rate: The interest on SSP100 for a unit time is called the rate of interest. It is expressed in percentage (%). The interest on SSP100 for 1 year is called rate per annum (abbreviated as rate % p. a.)
Simple interest is calculated only on the principal amount, or on that portion of the principal amount that remains. It excludes the effect of compounding. It is denoted by **S.I.**
The simple interest is calculated uniformly only on the original principal throughout the loan period.

SIMPLE INTEREST = $\dfrac{Principal \times Rate \times Time}{100}$

$$S.I = \frac{PRT}{100}$$

Where P = Principal, R = Rate and T = Time in years.
While calculating the time period between two given dates, the day on which the money is borrowed is not counted for interest calculations while the day on which the money is returned, is counted for interest calculations.

For converting the time in days into years, we always divide by 365, whether it is an ordinary year or a leap year.

Example 6.

Juma borrowed SSP15,000 from a financial institution for $2\frac{1}{2}$ years at simple interest of 12% P.a. How much interest did he pay?

Abbreviations	Words
I	Interest(money earned)
P	Principal(money borrowed or deposited)
R	Rate always represented as percentage
T	Time expressed in years
A	Amount = Interest plus principle

Formula $I = P\times\frac{R}{100}\times T$

Solution

P = SSP15,000

T = $2\frac{1}{2}$

R = 12%

$$I = 15{,}0\cancel{00}\times\frac{\overset{6}{\cancel{12}}}{\cancel{100}}\times\frac{5}{\cancel{2}}$$

Juma paid an interest of SSP 4,500

Interest paid was SSP 4,500

Amount Juma paid after $2\frac{1}{2}$ years was $P + I = A$

SSP 15,000 + SSP 4,500 = SSP19,500

Example 7.

Winfrey was issued a loan of SSP 500000 on simple interest. After 6 months, she paid back an interest of SSP 25000. At what interest rate was the loan issued?

Solution

$$S.I = \frac{PRT}{100}$$

$$25000 = \frac{500000 \times R \times 0.5}{100}$$

$$2500000 = 500000 \times R \times 0.5$$

$$R = \frac{2500000}{500000 \times 0.5}$$

$$= 10\% \text{ p.a}$$

Exercise 4.

1. Maryanne was issued with a loan of SSP 325,000 from a bank on simple interest at a rate of 18.5% p.a. how much money is she expected to pay the bank after 5 years?
2. David took a loan of SSP200,500 from a financial institution at a rate of 12.5% p.a. after what duration is he expected to pay an interest of SSP 100250?

To find Rate when Principal Interest and Time are given the rules are.
Interest = (Principal × Rate × Time)÷100
Rate = (100 × Interest) ÷ (Principal × Time)

Activity 2:

1. Find Rate, when Principal = SSP 3000; Interest = SSP 400; Time = 3 years.
2. Find Principal when Time = 4 years, Interest = SSP 400; Rate = 5% p.a.
3. Richard deposits 5400 and got back an amount of 6000 after 2 years. Find Richard's interest rate.

4. A farmer borrowed SSP 45,000 from a bank for buying a water pump. If she was charged a simple interest rate of 9%P.a. How much;

    a) Interest did she pay at the end of 18 months?

    b) Amount did she pay at the end of 18 months

5. Martin deposited SSP 90,000 in a bank account, which paid a simple interest rate at 10%. How much interest did he earn after 3 years?
6. Jemma borrowed SSP120,000 from a bank that charged simple interest at the rate of 15%. How much should she pay back the bank at the end of two years?
7. Shahin deposited SSP10,000 for a period of two years. She was charged simple interest at the rate of 15% per year. How much interest did she get?
8. Hussein deposited SSP100,000 in a financial institution that offered simple interest at the rate of 5% per annum. How much interest had Hussein's money earned after $1\frac{1}{2}$ years?

6.6 Compound interest in South Sudanese Pounds

Compound interest is the interest paid on the original principal and on the accumulated past interest.

When you borrow money from a bank, you pay interest. Interest is a fee charged for borrowing the money, it is a percentage charged on the principal amount for a period of a year.

Compound Interest (C.I.) = Final Amount – Original Principal

When calculating compound interest, the amount for the first year is used as principal for the second year. The time (T) is always 1.

Example 8.

Calculate the amount and the compound interest on SSP 12000 for 2 years at 5% per annum compounded annually.

Solution: For 1st year: P = SSP 12000; R=5% and T=1 year

Therefore; Interest $= \frac{PRT}{100}$

$$= \frac{12000 \times 5 \times 1}{100}$$

$$= 600$$

Amount = 12000 + 600 = SSP 12600

For 2nd year: P = SSP12600; R=5% and T=1year

$$\text{Therefore; Interest} = \frac{PRT}{100}$$

$$= \frac{12600 \times 5 \times 1}{100}$$

$$= 630$$

Amount = 12600 + 630 = SSP 13230

Example 9.

Luka deposited SSP10,000 in a bank which paid a compound interest at the rate of 12% per annum. If he withdrew all the money after two years, how much money did he withdraw?

Solution

Method 1:

1st year interest $= P\times\frac{R}{100}\times T = Sh100\cancel{00}\times\frac{12}{\cancel{100}}\times 1 = Sh1200$

Principal 2nd year =SSP10,000 + SSP1,200 = SSP11,200

2nd year interest $= SSP11{,}200\times\frac{12}{100}\times 1 = SSP1{,}344$

Total interest = SSP1,200 + SSP1,344 = SSP2,544

Total amount withdrawn = Deposit + Total Interest

Total amount withdrawn = SSP10,000 + SSP2,544 =SSP12,544

=SSP12,544

Method 2:

$$I = \frac{P\times R\times T}{100}$$

$$CI = P\times\frac{R_1}{100}\times\frac{R_2}{100}\times T$$

$$CI = 10{,}000\times\frac{112}{100}\times\frac{112}{100}\times 1 = 12{,}544$$

Where R_1= 1st year

R_2= 2nd year

C1= Compound interest

T= Time

The time (T) is multiplied by 1 since we are squaring the rate

∴ The compound interest for the two years is $SSP2{,}544 = 12{,}544 - 10{,}000$

Total amount withdrawn is SSP 2,544 (compound interest) plus initial deposit (principal) SSP 10,000.

The answer is SSP 12,544.

Exercise 5.

1. Susan borrowed SSP 200,000 from a money lender for a period of two years at a compound interest rate of 8% per year. How much did she pay all together?
2. Stephen borrowed SSP 250,000 from a bank for a duration of that charged a compound interest rate of $12\frac{1}{2}\%$ P.a. How much money should he pay the bank at the end of two years?
3. A trader deposited SSP18,000 for 2 years in a bank paying compound interest at the rate of 8% P.a. How much did she save in her account at the end of 3 years?
4. John borrowed SSP 40,000 from a bank which he paid a compound interest at the rate of $7\frac{1}{2}\%$ P.a. What was the total interest at the end of the second year?
5. Abdi deposited SSP 480,000 in a bank that paid a compound interest at the rate of 12% P.a. How much money did he pay back after $1\frac{1}{2}$ years?
6. Isaac deposited SSP100,000 in a financial institution that paid compound interest at the rate of 20%P.a. How much did he get at the end of the third year?
7. Stella deposited SSP 200,000 in a bank that paid a compound interest rate of 12% P.a. How much money was in her account at the end of two years?
8. Samuel borrowed SSP150,000 for a period of two years. He was charged compound interest at the rate of 12% per year. How much interest did he pay altogether?
9. Zachariah deposited his savings in a bank which paid a simple interest at a rate of 15% P.a. for a period of 3 years, while Oliver deposited the same amount of SSP 45,000 in saving account which pays a compound interest of 10%P.a. at the end of three years both withdrew the deposits with the interest, who was paid more than the other and by how much?

Activity 3:

With the guidance of a teacher, visit a local businessperson and listen to him/her to explain how they run their business and what sort of accounts they keep.

Example 10.

Calculate the amount and the compound interest on SSP 8000 for years at 10% per annum compounded yearly for 1 ½ years.

Solution

Interest $= \frac{PRT}{100}$

$= \frac{8000 \times 10 \times 1}{100}$

$= 800$

Amount = 8000 + 800 = SSP 8800

For 2nd year: P = SSP 8800; R=10% and T=0.5 year

Interest $= \frac{PRT}{100}$

$= \frac{8800 \times 10 \times 0.5}{100}$

= SSP 440

Amount = 8800 + 440 = SSP 9240

Compound interest = Amount – principal

= 9240 – 8000

= SSP1240

Exercise 6.

1. Racheal borrows SSP 12,000 at 10% per annum interest compounded half-yearly. Calculate the total amount she has to pay at the end of 30 months in order to clear the entire loan.
2. On a certain sum of money, invested at the rate of 10% per annum compounded annually, the interest for the first year plus the interest for the second year is SSP 2652. Find the sum.
3. A sum of money is lent at 8% per annum compound interest. If the interest for the second year exceeds that for the first year by SSP 96, find the sum of money.
4. A person invested SSP 8000 every year at the beginning of the year, at 10% per annum compounded interest. Calculate his total savings at the beginning of the third year.
5. A sum of SSP13500 is invested at 16% per annum compound interest for 5 years.

   Calculate:

   i. interest for the first year
   ii. the amount at the end of the first year
   iii. Interest for the second year.
6. Jackline borrowed SSP 7500 from Risper at 8% per annum compound interest. After 2 years she gave SSP 6248 back and a TV set to clear the debt. Find the value of the TV set.
7. It is estimated that every year, the value of the asset depreciates at 20% of its value at the beginning of the year. Calculate the original value of the asset if its value after two years is SSP 10240.
8. Find the sum that will amount to SSP 4928 in 2 years at compound interest, if the rates for the successive year are at 10% and 12% respectively.
9. Joan opens up a bank account on 1st Jan 2010 with SSP 24000. If the bank pays 10% per annum and the person deposits SSP 4000 at the end of each year, find the sum in the account on 1st Jan 2012.

6.7 Cash accounts

Example 11.

On 1^{st} January 2017, Mark had a capital of SSP17000. On 5^{th} January he bought pawpaws for SSP2400. On 7th January, he bought oranges for SSP 1000 and mangoes for SSP 2000.
By 10^{th} January, he had sold pawpaws for SSP 5000, oranges for SSP 2400 and mangoes for SSP 4000.
On the same day (10^{th}January) he paid SSP 700 for transport and SSP500 a market fee.

- ❖ Prepare mark's market cash account as at 11^{th}January and balance it.
- ❖ What was his balance on 11^{th} January 2017?
- ❖ What was his profit?

Solution

MARK'S MARKET CASH ACCOUNT

DATE	CASH IN			CASH OUT	
	Particulars	SSP	Date	Particulars	SSP
2017	Capital	17000	2017	Purchase	
Jan.1	Pawpaws sale	5000	Jan.5	Pawpaws	2400
Jan.10	Oranges sale	2400	Jan.7	Oranges	1000
Jan.10	Mangoes sale	4000	Jan.7	Mangoes	2000
Jan.10			Jan.10	Transport	700
			Jan.10	Market fee	500
				Balance (cash in hand)	21800
		28400			28400
Jan.11	Balance	21800			

Profit = *Balance (cash in hand) - capital (or balance at the start of business)*

SSP21800 - SSP17000

=SSP4800

WORKING PROCEDURE

1. The layout is divided into two sides:
   - Left hand side (cash in) for all the money received.
   - Right hand side (cash out) for all money spent
2. Each of the sides has 3 columns:
   - Date
   - particulars
   - money (SSP)

a. **Date:** The year is written at the top of the date column .Each of the particulars goes with its date.

b. **Particulars:** These are written in short phrases e.g. orange sale.

c. **Money:** Do all calculations on a separate piece of paper.

<u>How to balance a cash account.</u>

Step 1: Find the sum in the left hand column.

(SSP17000+SSP5000+SSP2400+SSP4000) = SSP28400

Enter this sum in the left hand side as shown.

Step 2: Find the sum of the expenditure, on a separate piece of paper.

(SSP2400+SSP1000+SSP2000+SSP700+SSP500)=SSP6600.

Do not enter this sum.

Step3: Subtract the total expenditure.

(SSP6600) from the total cash in (SSP28400) to get the balance (cash in hand) SSP 21800.

Step 4: Enter the balance (cash in hand) SSP21800 in the right hand side as shown.

Step 5: Find the sum in the right hand side by adding the total expenditure (SSP6600) and the balance or cash in hand SSP21800.

i.e. SSP6600+SSP21800=SSP28400.

NOTE: the sum in the right hand side should be equal to the sum in the left hand side. If the sum in the left hand side and the right hand side are equal, you have balanced the account. If they are not equal, then you have not succeeded in balancing the account

Exercise 7.

In groups, prepare and balance cash account for the different accounts:

1. Shopkeepers account.
   On first April 2017, a shopkeeper had a cash balance of SSP 49500 in hand. On fifth April, a bill of SSP 5990 was paid to flour mills limited. He received SSP20000 for goods sold in week ending 17th April and SSP35850 in a week ending 14th April. On 22nd April he paid SSP45600 to a bread company and SSP12350 to seed company he received SSP62300 for goods sold in the week ending 22nd April and SSP53400 for goods sold in the week ending 29th April. On 30th April he paid SSP10000 rent, SSP850 for lighting and SSP4500 wages and deposited SSP 20000 in his bank account.
   a. Prepare this shopkeepers cash account and balance it.
   b. What was his balance in his cash account as at 1st may 2017?
2. Carpenters account:
   A carpenter had a balance in his hand of SSP17800. On 1st jan.2016, on 15th Jan, he spent SSP5900 on wood, SSP680 on nails and SSP8990 on tools. On 21st Jan. he sold 10 chairs at SSP2400 each and 6 tables at each SSP4000 .on 27th Jan. he spent SSP 11900 on nails. He transferred SSP9990 to his bank account and paid his labourers a total of SSP 7900 on 31st Jan.
   a. Prepare the carpenters cash account.
   b. What was the balance in his cash account as at 1st Feb 2016?
3. Poultry account.
   A poultry farmer had a flock of 600 layers and a cash balance of SSP26000 on 1st June 2015. On 2nd June, he bought 3 sacks of layers marsh at SSP 2500 each on 5th June, he sold 90 trays of eggs at SSP 300 each and bought 6 sacks layers mash at SSP 2500 each.
   On 21st June he sold 178 trays of eggs at SSP 300 a tray and bought 10 sacks of layers marsh at SSP 1500 a sack.
   On 28th June he sold 100 trays of eggs at SSP 300 a tray.
   On 29th June, he bought 80 egg trays at SSP50 each and paid his worker SSP 4600 on 30th June
   a. Prepare and balance the poultry cash account.
   b. If the farmer banked the balance, how much money did he bank?
   c. How much money did he earn from his poultry farming during the month of June 2015?